

**TEST - 15****ANSWERS****PHYSICS**

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30. (4)

**CHEMISTRY**

31. (1)
32. (2)
33. (4)
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35. (1)
36. (3)
37. (4)
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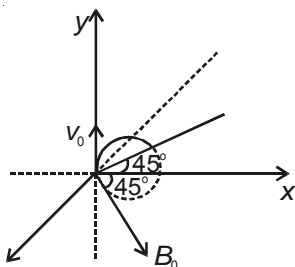
**MATHEMATICS**

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90. (4)

# Answers & Hints

## PART - A : (PHYSICS)

1. Answer (2)



at  $t = 0, \vec{v}_i = v_0 \hat{j}$

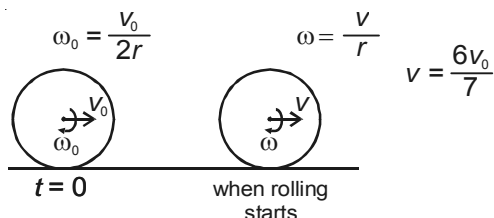
at  $t = \frac{\pi m}{B_0 q}, \vec{v}_f = -v_0 \hat{j}$

$$\Delta \vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\Delta \vec{p} = -2mv_0 \hat{j}$$

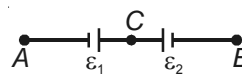
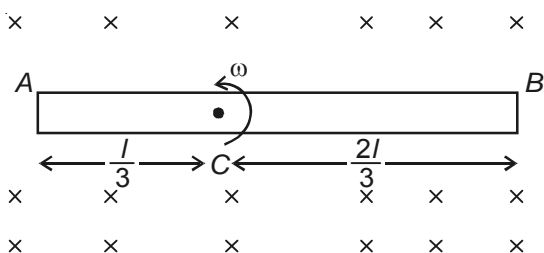
$$\vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t} \Rightarrow \vec{F}_{avg} = \left( \frac{-2qB_0 v_0}{\pi} \right) \hat{j}$$

2. Answer (2)



$$\begin{aligned} \text{Rotational work done by friction} &= \frac{1}{2} I \omega^2 - \frac{1}{2} I \omega_0^2 \\ &= \frac{1}{2} \times \frac{2}{5} m r^2 (\omega^2 - \omega_0^2) \\ &= \frac{19 m v_0^2}{196} \end{aligned}$$

3. Answer (1)



$$\epsilon_1 = \frac{1}{2} B \omega \frac{l^2}{9}$$

$$\epsilon_1 = \frac{B \omega l^2}{18}$$

$$\epsilon_2 = \frac{1}{2} B \omega \times \frac{4l^2}{9}$$

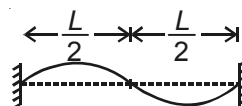
$$\epsilon_2 = \frac{4B \omega l^2}{18}$$

$$V_A - V_B = -\epsilon_1 + \epsilon_2 \Rightarrow V_A - V_B = \frac{B \omega l^2}{6}$$

4. Answer (3)

$$f = \frac{v}{L}$$

For  $n^{\text{th}}$  harmonic  $f_n = \frac{nv}{2L}$



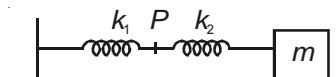
$$\Rightarrow \frac{v}{L} = \frac{nv}{2L} \Rightarrow n = 2$$

Particle at  $x = \frac{L}{8}$  lies in left loop.

Particle at  $x = \frac{5L}{8}$  lies in right loop.

So, phase difference between them is  $\pi$ .

5. Answer (1)



$$\text{Elongation in left spring } x_1 = \frac{k_2 x}{k_1 + k_2}$$

Where  $x$  is total elongation,

$$\text{So, Amplitude of vibration of } P \text{ is } \frac{k_2 A}{k_1 + k_2}$$

6. Answer (3)

$$PV^2 = \text{constant} \quad \text{---(1)}$$

$$B = -V \left( \frac{dP}{dV} \right) \quad \text{---(2)}$$

Differentiate eq. (1)

$$V^2 dP + P \times 2V dV = 0 \Rightarrow 2PV dV = -V^2 dP$$

$$\Rightarrow -V \left( \frac{dP}{dV} \right) = 2P \Rightarrow B = 2P$$

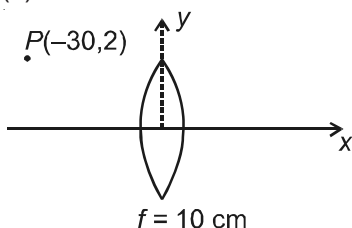
By ideal gas equation

$$PV = nRT \quad \text{---(3)}$$

From (1) & (3)

$$P \propto T^2 \Rightarrow B \propto T^2$$

7. Answer (2)



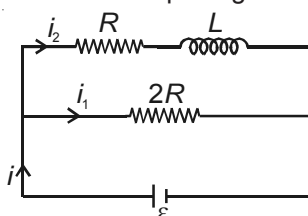
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{10} + \frac{1}{-30}$$

$$\Rightarrow v = 15 \text{ cm}, x_i = 15 \text{ cm}$$

$$\frac{y_i}{y_0} = \frac{v}{u} \Rightarrow y_i = \frac{15}{-30} \times 2 \Rightarrow y_i = -1 \text{ cm}$$

8. Answer (2)

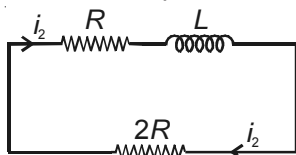
At steady state before opening the switch



$$i = i_1 + i_2$$

$$i_1 = \frac{\varepsilon}{2R}, i_2 = \frac{\varepsilon}{R}$$

Just after the switch is opened



$$i_2 R + L \frac{di_2}{dt} + i_2 \times 2R = 0$$

$$L \frac{di_2}{dt} = -3i_2 R$$

$$L \left( \frac{di_2}{dt} \right) = -3\varepsilon \Rightarrow \frac{di_2}{dt} = -\frac{3\varepsilon}{L}$$

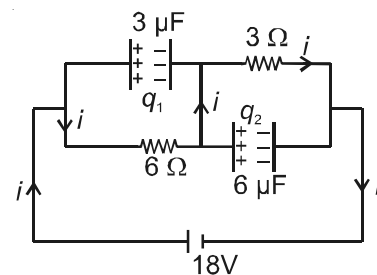
9. Answer (3)

$$i = 2A$$

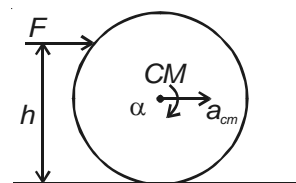
$$q_1 = 3 \times 12$$

$$q_1 = 36 \mu\text{C}$$

$$q_2 = 6 \times 6 \Rightarrow q_2 = 36 \mu\text{C}$$



10. Answer (4)



$$a_{cm} = \frac{F}{m} \quad \text{---(1)}$$

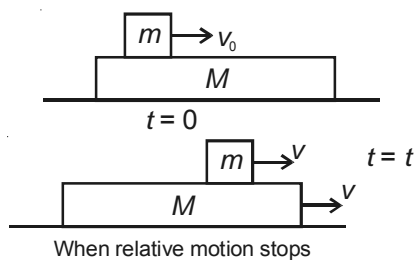
$$\alpha = \frac{F(h-r)}{I_{cm}}$$

$$\Rightarrow \alpha = \frac{5F(h-r)}{2mr^2} \quad \text{---(2)}$$

For rolling  $a_{cm} = r\alpha$

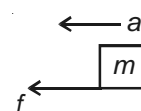
$$\Rightarrow \frac{F}{m} = \frac{5F(h-r)}{2mr} \Rightarrow h = \frac{7r}{5}$$

11. Answer (1)



By conservation of linear momentum

$$mv_0 = (M+m)v \Rightarrow v = \frac{mv_0}{M+m}$$



$$a_1 = \mu g$$

$$\bar{v} = \bar{u} + \bar{a}t$$

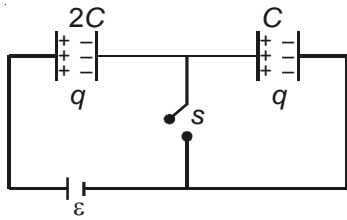
$$v = v_0 - \mu g t \Rightarrow \mu g t = v_0 - v$$

$$\Rightarrow \mu g t = v_0 - \frac{mv_0}{M+m}$$

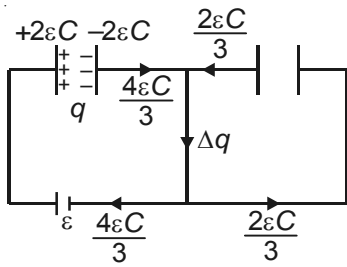
$$\Rightarrow t = \frac{mv_0}{(M+m)\mu g}$$

12. Answer (2)

$$q = \epsilon C_{eq} \Rightarrow q = \frac{2\epsilon C}{3}$$



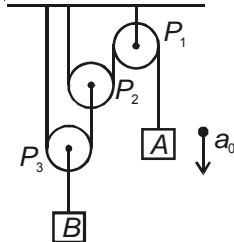
Charge flown through switch



$$\Delta q = \frac{4\epsilon C}{3} + \frac{2\epsilon C}{3}$$

$$\Delta q = 2\epsilon C$$

13. Answer (3)



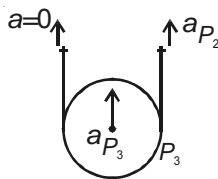
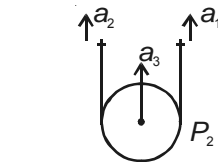
$$2a_3 = a_1 + a_2$$

$$a_3 = a_{P_2}, a_1 = a_0, a_2 = 0$$

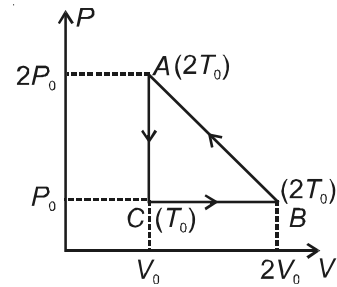
$$a_{P_2} = \frac{a_0}{2}$$

$$a_{P_3} = \frac{a_{P_2}}{2} \Rightarrow a_{P_3} = \frac{a_0}{4}$$

$$\Rightarrow a_B = \frac{a_0}{4} \text{ upward.}$$



14. Answer (4)



$$T_0 = \frac{P_0 V_0}{nR}$$

$$\Delta Q_{BC} = -\frac{5}{2} nRT_0 = -\frac{5}{2} P_0 V_0$$

$$\Delta Q_{CA} = \frac{3}{2} nRT_0 = \frac{3}{2} P_0 V_0$$

$$\Delta Q_{net} = W_{net}$$

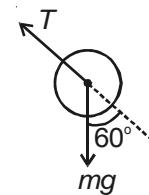
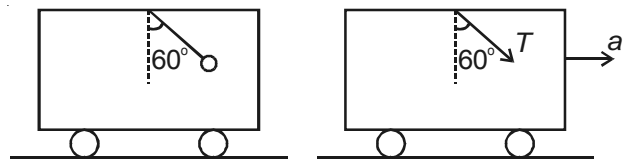
$$\Delta Q_{AB} = W_{net} - \Delta Q_{BC} - \Delta Q_{CA}$$

$$\Delta Q_{AB} = \frac{3}{2} P_0 V_0$$

$$\eta = \left[ 1 - \frac{|\Delta Q_{-ve}|}{|\Delta Q_{+ve}|} \right] \times 100 \Rightarrow \eta = \left[ 1 - \frac{5/2}{3} \right] \times 100$$

$$\Rightarrow \eta = \frac{50}{3} \Rightarrow \eta = 16.67\%$$

15. Answer (4)



$$T = mg \cos 60^\circ$$

$$\Rightarrow T = \frac{1}{2} mg$$

$$T \sin 60^\circ = 10m \times a \Rightarrow a = \frac{\sqrt{3}}{4} \text{ m/s}^2$$

16. Answer (3)

$$F \times l = I\alpha$$

$$\Rightarrow Mg \times l = \frac{1}{3} m l^2 \alpha \Rightarrow \alpha = \frac{3g}{l}$$

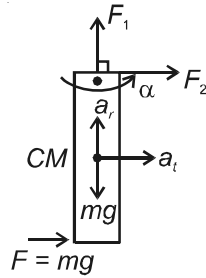
$$a_t = \frac{l}{2} \alpha \Rightarrow a_t = \frac{3g}{2}$$

$$a_r = \frac{l}{2} \omega^2 \Rightarrow a_r = 0$$

$$F_1 - mg = ma_r \Rightarrow F_1 = mg$$

$$F + F_2 = ma_t \Rightarrow F_2 = \frac{3}{2} mg - mg \Rightarrow F_2 = \frac{mg}{2}$$

$$\text{Net force by the hinge} = \sqrt{F_1^2 + F_2^2} = \frac{\sqrt{5}}{2} mg$$



17. Answer (4)

18. Answer (1)

19. Answer (3)

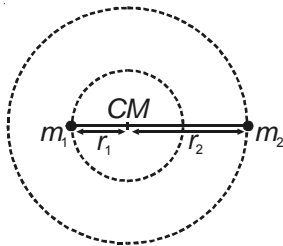
20. Answer (3)

21. Answer (2)

22. Answer (1)

$$r_1 + r_2 = r$$

$$r_1 = \frac{m_2 r}{m_1 + m_2} \text{ and } r_2 = \frac{m_1 r}{m_1 + m_2}$$



$$\frac{Gm_1 m_2}{r^2} = m_1 r_1 \omega^2$$

$$\Rightarrow \frac{Gm_1 m_2}{r^2} = m_1 \times \left( \frac{m_2 r}{m_1 + m_2} \right) \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{G(m_1 + m_2)}{r^3}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{G(m_1 + m_2)}{r^3}}$$

$$\Rightarrow T = 2\pi \sqrt{\frac{r^3}{G(m_1 + m_2)}}$$

23. Answer (3)

$$k = \frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2 \Rightarrow k = \frac{1}{2} (I_1 + I_2) \omega^2$$

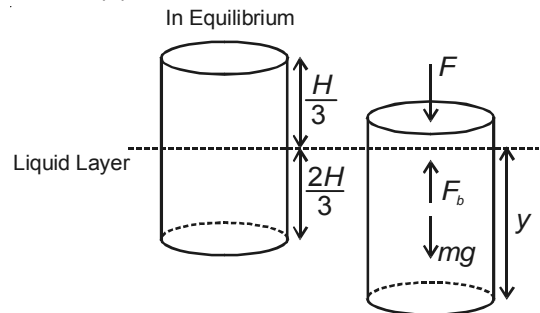
$$\Rightarrow k = \frac{1}{2} \left( \frac{m_1 m_2}{m_1 + m_2} \right) r^2 \omega^2 \Rightarrow k = \frac{Gm_1 m_2}{2r}$$

24. Answer (1)

$$E = K + U$$

$$= \frac{Gm_1 m_2}{2r} + \left( -\frac{Gm_1 m_2}{r} \right) \Rightarrow E = -\frac{Gm_1 m_2}{2r}$$

25. Answer (2)



At equilibrium

$$AH\sigma g = A \times \frac{2H}{3} \times \rho g$$

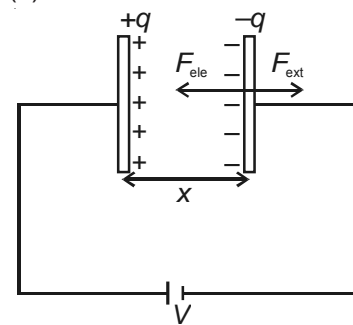
$$\Rightarrow \sigma = \frac{2}{3} \rho$$

$$F + mg = F_b \Rightarrow F = F_b - mg$$

$$\Rightarrow F = yA\rho g - HA\sigma g \Rightarrow F = Ag\rho \left( y - \frac{2H}{3} \right)$$

$$dW = Fdy \Rightarrow W = \int_{2H/3}^H Fdy \Rightarrow W = \frac{AH^2 \rho g}{12}$$

26. Answer (3)



$$C = \frac{\epsilon_0 A}{x} \quad q = \epsilon V \Rightarrow q = \frac{\epsilon_0 AV}{x}$$

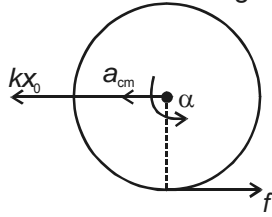
$$F_{ele} = \frac{q^2}{2\epsilon_0 A} \Rightarrow F_{ext} = F_{ele}$$

$$\Rightarrow F_{ext} = \frac{\epsilon_0 AV^2}{2x^2} \Rightarrow dW_{ext} = F_{ext} dx$$

$$W_{ext} = \frac{\epsilon_0 AV^2}{2} \int_d^{2d} \frac{dx}{x^2} \Rightarrow W_{ext} = \frac{\epsilon_0 AV^2}{4d}$$

27. Answer (1)

At maximum elongation



$$\frac{1}{2} kx_0^2 = \frac{1}{2} I\omega_0^2 + \frac{1}{2} mv_0^2$$

$$\frac{1}{2} kx_0^2 = \frac{3}{4} mv_0^2 \Rightarrow x_0 = \sqrt{\frac{3m}{2k}} v_0$$

$$a_{cm} = \frac{kx_0 - f}{m} \quad \text{---(1)}$$

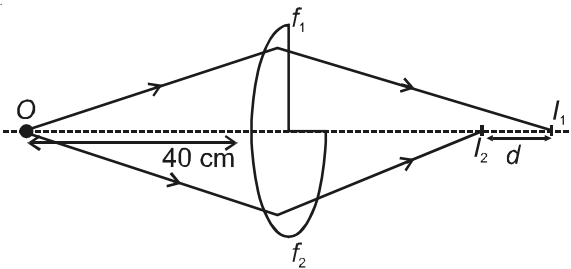
$$\alpha = \frac{2f_x r}{mr^2} \Rightarrow \alpha = \frac{2f}{mr} \quad \text{---(2)}$$

For rolling  $a_{cm} = r\alpha \Rightarrow \frac{2f}{mr} = \frac{kx_0 - f}{m} \Rightarrow f = \frac{kx_0}{3}$

$$\mu N \geq f \Rightarrow \mu mg \geq \frac{kx_0}{3} \Rightarrow \mu mg \geq \sqrt{\frac{mk}{6}} v_0$$

$$\Rightarrow \mu \geq \sqrt{\frac{k}{6m}} \frac{v_0}{g} \Rightarrow \mu_{\min} = \sqrt{\frac{k}{6m}} \left( \frac{v_0}{g} \right)$$

28. Answer (3)



Focal length of upper half part of lens

$$\frac{1}{f_1} = (1.5 - 1) \left[ \frac{1}{10} - \frac{1}{\infty} \right] \Rightarrow f_1 = 20 \text{ cm}$$

Focal length of lower half part of lens

$$\frac{1}{f_2} = (1.5 - 1) \left[ \frac{1}{10} - \frac{1}{-10} \right] \Rightarrow \frac{1}{f_2} = \frac{0.5 \times 2}{10}$$

$$\Rightarrow f_2 = 10 \text{ cm}$$

Image formation by upper half of lens

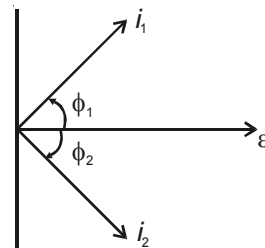
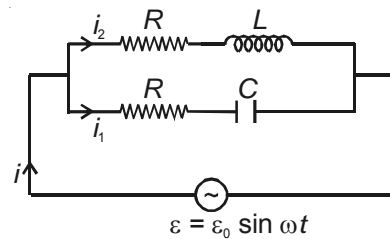
$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} + \frac{1}{-40} \Rightarrow v_1 = 40 \text{ cm}$$

Image formation by lower half of lens

$$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{10} - \frac{1}{-40} \Rightarrow v_2 = \frac{40}{3}$$

$$d = 40 - \frac{40}{3} \Rightarrow d = \frac{80}{3} \text{ cm}$$

29. Answer (1)

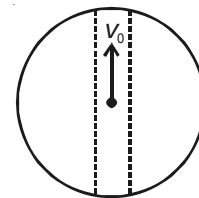


Net current is in phase of e.m.f., if

$$\frac{\epsilon_0}{Z_1} \sin \phi_1 = \frac{\epsilon_0}{Z_2} \sin \phi_2 \Rightarrow \frac{1}{Z_1} \times \frac{x_C}{Z_1} = \frac{1}{Z_2} \times \frac{x_L}{Z_2}$$

$$\Rightarrow \frac{x_C}{R^2 + x_C^2} = \frac{x_L}{R^2 + x_L^2} \quad R = \sqrt{\frac{L}{C}}$$

30. Answer (4)



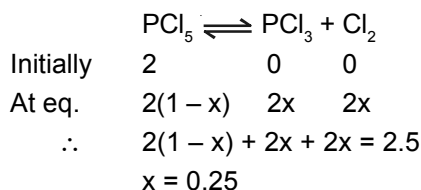
By conservation of mechanical energy

$$\frac{1}{2} mv_0^2 - \frac{3GMm}{2R} = 0 + 0$$

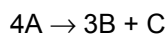
$$\Rightarrow v_0 = \sqrt{\frac{3GM}{R}}$$

## PART - B : (CHEMISTRY)

31. Answer (1)



32. Answer (2)



$$-\frac{1}{4} \frac{d[A]}{dt} = \frac{1}{3} \frac{d[B]}{dt} = \frac{d[C]}{dt}$$

$$\frac{1}{4} K_1 [A]^4 = \frac{1}{3} K_2 [A]^4 = K_3 [A]^4$$

$$\frac{K_1}{4} = \frac{K_2}{3} = K_3$$

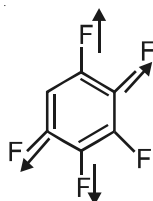
$$\therefore 3K_1 = 4K_2 = 12K_3$$

33. Answer (4)

34. Answer (2)

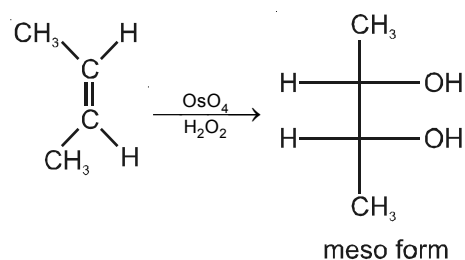
 $\Delta S > 0$  for irreversible process

35. Answer (1)



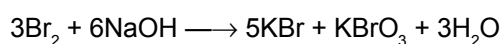
It becomes equal to fluorobenzene because effect of four fluorine atom cancels to each other.

36. Answer (3)

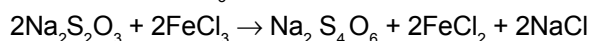
 $\text{OsO}_4/\text{H}_2\text{O}_2$  gives syn addition


37. Answer (4)

It is a disproportionation reaction



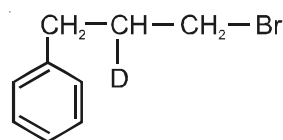
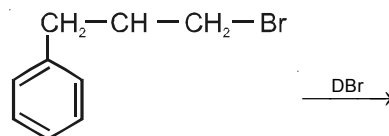
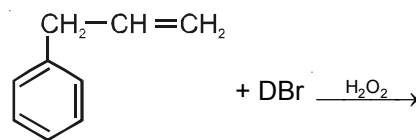
38. Answer (3)

 Reduction of  $\text{FeCl}_3$ 


39. Answer (4)

Number of unpaired electrons are changing

40. Answer (2)



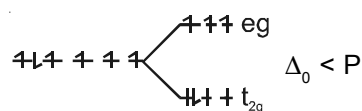
41. Answer (1)

 $n$ -factor of  $\text{NH}_3 = 3$ ,  $M_1 = 3E_1$ 
 $n$ -factor of  $\text{N}_2 = 6$ ,  $M_2 = 6E_2$ 

$$\therefore M_1 - M_2 = 3E_1 - 6E_2$$

42. Answer (4)

In weak field octahedral complex

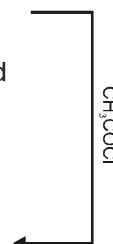
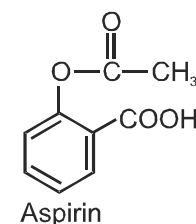
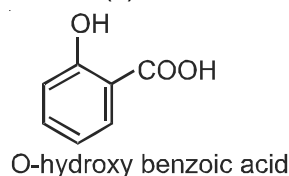

 $d$ -orbital

$$d^6 \rightarrow t_{2g}^4 e_g^2$$

$$\therefore \text{Difference} = 4 - 2 = 2$$

43. Answer (1)

44. Answer (3)



45. Answer (3)

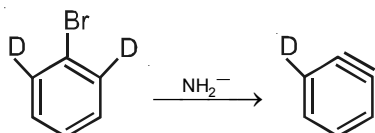
I = Cis-Trans-cis-cis-trans isomer of [10] annulene

II = all cis-isomer of [10] annulene

46. Answer (3)

Cr metal is commercially extracted by Al reduction method.

47. Answer (1)



48. Answer (2)

49. Answer (3)

At high temperature, nucleophilic addition can occur in olefins having -I group attracted to olefinic carbon.

50. Answer (1)

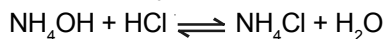
 $\text{CH}_3\text{COOH}$  dimerizes in benzene hence molecular weight is 2 times.

51. Answer (1)

52. Answer (3)

NaCl is the salt of strong acid and strong base. Therefore, no hydrolysis take place and solution will be neutral.

53. Answer (2)

 $\therefore$  50%  $\text{NH}_3$  is neutralised

$$[\text{NH}_4^+] = [\text{NH}_3]$$

$$\therefore \text{p}^{\text{OH}} = \text{p}^{\text{K}_b} + \log \frac{[\text{NH}_4^+]}{[\text{NH}_3]}$$

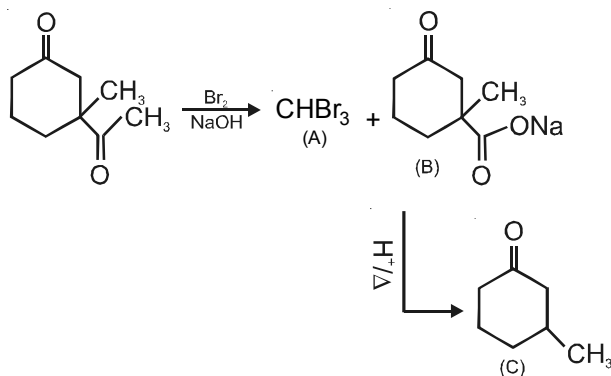
$$\text{p}^{\text{OH}} = 4.72 + \log 1$$

$$\therefore \text{pH} = 14 - 4.72$$

$$\text{pH} = 9.28$$

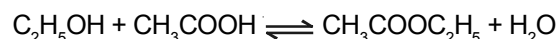
54. Answer (4)

55. Answer (4)



56. Answer (1)

57. Answer (1)



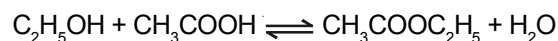
$$\text{At } t = 0 \quad 1 \quad 1 \quad 0 \quad 0$$

$$t_{\text{eqn.}} \quad 1-x \quad 1-x \quad x \quad x$$

$$\therefore x = 2/3$$

$$\Rightarrow (1 - 2/3) \quad (1 - 2/3) \quad 2/3 \quad 2/3$$

$$\therefore K_c = \frac{\frac{2}{3} \times \frac{2}{3}}{\frac{1}{3} \times \frac{1}{3}} = 4$$



$$t = 0 \quad 1 \quad 1 \quad 0 \quad 0$$

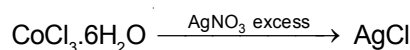
$$t_{\text{eqn.}} \quad (2-x) \quad 2-x \quad x \quad x$$

$$\therefore K_c = \frac{x^2}{(2-x)^2}$$

$$4 = \frac{x^2}{(2-x)^2}$$

$$\therefore x = 1.33$$

58. Answer (3)



0.1 M

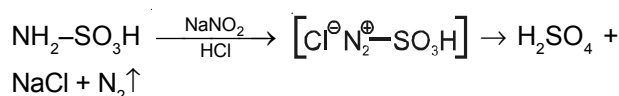
Weight of AgCl =  $n \times 0.01 \times 143.5$ 

$$n = \frac{2.87}{0.01 \times 143.5}$$

= 2 (number of AgCl molecules)

 $\therefore$  The formula of complex is  $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}] \text{Cl}_2 \cdot \text{H}_2\text{O}$ 

59. Answer (2)



60. Answer (1)

$$\Delta G = -\Delta H - T(+\Delta S)$$

$$\Delta G = -ve$$

PART - C : (MATHEMATICS)

61. Answer (1)

It is given that  $\sin\theta_1 + \operatorname{cosec}\theta_1 + \tan\theta_2 + \cot\theta_2 = 4$

$$\Rightarrow \theta_1 = \frac{\pi}{2} \text{ and } \theta_2 = \frac{\pi}{4}$$

so,  $\tan\theta_2 = 1$

$$\Rightarrow 1 = \frac{2 \tan\left(\frac{\theta_2}{2}\right)}{1 - \tan^2\left(\frac{\theta_2}{2}\right)}$$

$$\Rightarrow \tan^2\left(\frac{\theta_2}{2}\right) + 2 \tan\left(\frac{\theta_2}{2}\right) - 1 = 0$$

$$\Rightarrow \tan\left(\frac{\theta_2}{2}\right) \text{ is a root of equation } x^2 + 2x - 1 = 0$$

62. Answer (4)

$$I = \int x^2 d(e^{2x})$$

$$I = \int x^2 \frac{d}{dx}(e^{2x}) dx$$

$$I = 2 \int x^2 \cdot e^{2x} dx$$

$$I = \frac{1 \cdot e^{2x}}{2} [2x^2 - 2x + 1] + C$$

63. Answer (4)

Let  $S = 1 + 2\alpha + 3\alpha^2 + \dots + n\alpha^{n-1}$

$$\alpha S = \alpha + 2\alpha^2 + \dots + n\alpha^n$$

$$\Rightarrow S(1 - \alpha) = 1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} - n\alpha^n$$

$$\Rightarrow S = \frac{1 - \alpha^n}{(1 - \alpha)^2} - \frac{n\alpha^n}{1 - \alpha} \text{ but } \alpha^n = 1$$

$$\text{so } S = \frac{-n}{1 - \alpha}$$

64. Answer (4)

The minimum vertical distance between the graphs of given curves

$$= \min(2 + \cos x - \sin x)$$

$$= \min\left[2 + \sqrt{2} \cos\left(x + \frac{\pi}{4}\right)\right]$$

$$= 2 - \sqrt{2}$$

65. Answer (4)

Orthocenter of points  $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$  is given

$$\text{by } \left(-\frac{1}{abc}, -abc\right)$$

66. Answer (4)

$$\therefore T_{m+1} = \frac{1}{1 - T_m}$$

$$\Rightarrow T_2 = \frac{1}{1 - T_1}$$

$$T_3 = \frac{1}{1 - T_2}$$

$$\text{So, } T_3 = \frac{1}{1 - \frac{1}{1 - T_1}}$$

$$T_3 = \frac{1 - T_1}{-T_1}$$

$$T_3 = \frac{T_1 - 1}{T_1} \text{ but } T_3 = T_1$$

$$\text{So, } T_1 = \frac{T_1 - 1}{T_1} \Rightarrow T_1^2 - T_1 + 1 = 0$$

Hence,  $T_1 = T_3 = T_5 = \dots T_{2011} = -\omega$  or  $-\omega^2$   
then  $(-\omega)^{2010}$  or  $(-\omega^2)^{2010} = 1$

67. Answer (1)

$$\therefore [\vec{a} \times \vec{b} \vec{b} \times \vec{c} \vec{c} \times \vec{a}] = [\vec{a} \vec{b} \vec{c}]^2$$

$$\text{and } [\vec{a} + \vec{b} \vec{b} + \vec{c} \vec{c} + \vec{a}] = 2[\vec{a} \vec{b} \vec{c}]$$

$$\text{and } 2[\vec{a} - \vec{b} \vec{b} - \vec{c} \vec{c} - \vec{a}] = 0$$

so given equation can be written as

$$32x^2 + 8x = 0$$

$$\Rightarrow x = 0, -\frac{1}{4}$$

68. Answer (4)

Equation of director circle to circle  $x^2 + y^2 + 2gx + 2fy + c = 0$  is given by  $x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$

69. Answer (4)

Equation of tangent to  $y^2 = 8x$  is  $y = mx + \frac{2}{m}$

Let  $(h, k)$  be the point of intersection

$$\Rightarrow k = mh + \frac{2}{m}$$

$$\Rightarrow m^2h - mk + 2 = 0$$

$$\Rightarrow m_1 + m_2 = \frac{k}{h}, m_1m_2 = \frac{2}{h}$$

$$\tan\alpha + \tan\beta = \frac{k}{h}$$

$$\tan\alpha \tan\beta = \frac{2}{h}$$

Given that  $\cot\alpha + \cot\beta = 2$

$$\Rightarrow \frac{\tan\alpha + \tan\beta}{\tan\alpha \tan\beta} = c$$

$$\Rightarrow \frac{\frac{k}{h}}{\frac{2}{h}} = c$$

$$\Rightarrow k = 2c$$

$$\Rightarrow y = 2c$$

Which is a straight line parallel to x-axis

70. Answer (1)

$$\frac{dy}{dx} = \frac{2x - 3y + 7}{3x - 2y + 5}$$

$$\Rightarrow 3x dy + 3y dx - 2y dy - 2x dx + 5dy - 7dx = 0$$

$$\Rightarrow 3d(xy) - 2y dy - 2x dx + 5dy - 7dx = 0$$

$$\Rightarrow 3 \int d(xy) - 2 \int y dy - \int 2x dx + 5 \int dy -$$

$$7 \int dx = 0$$

$$\Rightarrow 3xy - y^2 - x^2 + 5y - 7x + C = 0$$

when  $x = 0, y = 1$

$$C = -4$$

$$\text{so, } x^2 + y^2 + 7x - 5y - 3xy + 4 = 0$$

71. Answer (2)

$$\text{Let } y = \frac{mx+p}{n}, \bar{y} = \frac{1}{n} (m\bar{x} + p)$$

$$\Rightarrow y - \bar{y} = \frac{m}{n} (x - \bar{x})$$

$$\Rightarrow \frac{1}{N} \Sigma (y - \bar{y})^2 = \frac{m^2}{n^2} \frac{1}{N} \Sigma (x - \bar{x})^2$$

$$\therefore \text{S.D of } y = \sqrt{\frac{m^2}{n^2} \frac{1}{N} \Sigma (x - \bar{x})^2}$$

$$= \sqrt{\frac{m^2}{n^2} \sigma^2}$$

$$\Rightarrow \left| \frac{m}{n} \right| \sigma$$

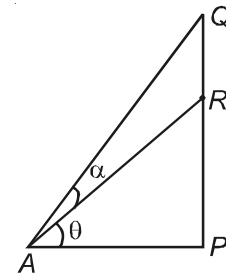
72. Answer (3)

Let  $\angle RAP = \theta$

$$\tan\theta = \frac{RP}{AP}$$

$$\tan\theta = \frac{RP}{nPQ}$$

$$\tan\theta = \frac{2RP}{2nPQ}$$



$$\tan\theta = \frac{PQ}{2nPQ} \Rightarrow \tan\theta = \frac{1}{2n}$$

Now in  $\Delta AQP$

$$\tan(\alpha + \theta) = \frac{PQ}{AP} = \frac{PQ}{nPQ} = \frac{1}{n}$$

$$\text{so, } \alpha = (\alpha + \theta) - \theta$$

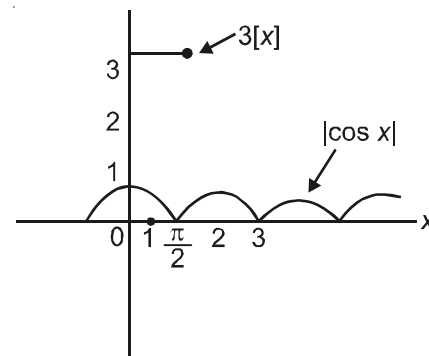
$$\Rightarrow \tan\alpha = \frac{\tan(\alpha + \theta) - \tan\theta}{1 + \tan(\alpha + \theta)\tan\theta}$$

$$\Rightarrow \tan\alpha = \frac{\frac{1}{n} - \frac{1}{2n}}{1 + \frac{1}{n} \cdot \frac{1}{2n}}$$

$$\Rightarrow \tan\alpha = \frac{n}{2n^2 + 1}$$

73. Answer (4)

Plot the graph of  $|\cos x|$  and  $3[x]$



Clearly no solution exists

74. Answer (1)

$$\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$$

$$\Rightarrow x = y = z = 1$$

75. Answer (4)

Equation of line passing through  $A(2, -3, -1)$  and  $B(8, -1, 2)$  is

$$\frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3} \Rightarrow \frac{x-2}{\frac{6}{7}} = \frac{y+3}{\frac{2}{7}} = \frac{z+1}{\frac{3}{7}}$$

The coordinates of point on this line at a distance 21 from A are

$$\left(2 + \frac{6 \cdot 21}{7}, -3 + \frac{2 \cdot 21}{7}, -1 + \frac{3 \cdot 21}{7}\right)$$

$$(20, 3, 8)$$

76. Answer (4)

We have,

$$n(S) = 100$$

$(E) = \{\text{even number, square of odd number}\}$

$$= \{2, 4, 6, 8, 10, \dots, 100, 1, 9, 25, 49, 81\}$$

$$n(E) = 55$$

$$\text{Required probability} = \frac{55}{100} = \frac{11}{20}$$

77. Answer (1)

78. Answer (4)

79. Answer (1)

80. Answer (4)

81. Answer (2)

**Answer of Q.No. 82 to 84**

82. Answer (1)

$$\text{Area of } \triangle APB = 2 \times \frac{1}{2} PM \times AM$$

$$\text{but } PM = PA \cos \theta$$

$$\text{so, area} = \sqrt{S_1} \cdot \frac{\sqrt{S_1}}{\sqrt{S_1 + r^2}} \cdot \frac{r\sqrt{S_1}}{\sqrt{S_1 + r^2}}$$

$$= \frac{r \cdot S_1^{3/2}}{S_1 + r^2}$$

83. Answer (2)

In  $\triangle PAO$

$$\tan \theta = \frac{OA}{PA} = \frac{r}{\sqrt{S_1}}$$

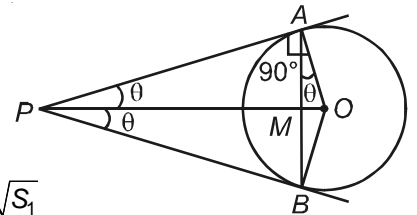
$$\text{Now } \tan 2\theta = \frac{\frac{2r}{\sqrt{S_1}}}{1 - \frac{r^2}{S_1}}$$

$$\tan 2\theta = \frac{2r\sqrt{S_1}}{S_1 - r^2}$$

$$\Rightarrow 2\theta = \tan^{-1} \frac{2r\sqrt{S_1}}{S_1 - r^2}$$

Angle between tangents

$$= \tan^{-1} \frac{2r\sqrt{S_1}}{S_1 - r^2}$$



84. Answer (2)

$$\text{Now } \cos \theta = \frac{AM}{AO}$$

$$AM = AO \cos \theta$$

$$AM = r \cos \theta$$

$$AM = \frac{r\sqrt{S_1}}{\sqrt{S_1 + r^2}}$$

$$AB = 2AM$$

$$AB = \frac{2r\sqrt{S_1}}{\sqrt{S_1 + r^2}}$$

85. Answer (3)

We observe that,

$${}^{2010}C_n \cdot {}^nC_r = \frac{2010!}{n!(2010-n)!} \cdot \frac{n!}{r!(n-r)!}$$

$$= \frac{2010!}{r!(2010-r)!} \times \frac{(2010-r)!}{(n-r)!(2010-n)!}$$

$$= {}^{2010}C_r \cdot {}^{2010-r}C_{n-r}$$

$$\text{Now } \sum_{n=r}^{2010} {}^{2010}C_r \cdot {}^{2010-r}C_{n-r}$$

$$\begin{aligned}
 &= {}^{2010}C_r \sum_{n=r}^{2010} \binom{2010-r}{n-r} \\
 &= {}^{2010}C_r 2^{2010-r} \therefore \sum_{r=1}^n {}^n C_r = 2^n \\
 \text{Again } &\sum_{r=1}^{2010} {}^{2010}C_r \times 2^{2010-r} \\
 &= 2^{2010} \sum_{r=1}^{2010} \frac{{}^{2010}C_r}{2^r} \\
 &= 2^{2010} \left[ \left(1 + \frac{1}{2}\right)^{2010} - 1 \right] \\
 &= 2^{2010} \left[ \left(\frac{3}{2}\right)^{2010} - 1 \right] \\
 &= 3^{2010} - 2^{2010}
 \end{aligned}$$

86. Answer (2)

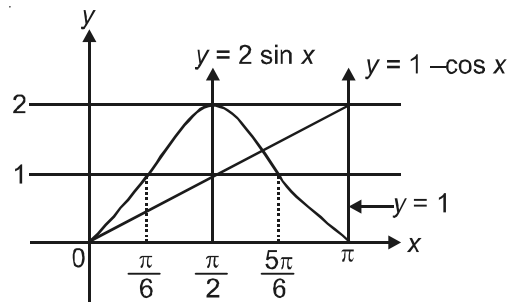
$$\begin{aligned}
 (1 + x + x^2)^n &= p_0 + p_1x + p_2x^2 + p_3x^3 + \dots + p_{2n}x^{2n} \\
 \text{Put } x &= 1, \omega, \omega^2 \text{ respectively} \\
 3^n &= p_0 + p_1 + p_2 + p_3 + \dots + p_{2n} \dots (1) \\
 0 &= p_0 + p_1\omega + p_2\omega^2 + p_3 \dots + p_{2n}\omega^{2n} \dots (2) \\
 0 &= p_0 + p_1\omega^2 + p_2\omega^4 \dots (3) \\
 \therefore 1 + \omega + \omega^2 &= 0 \\
 \text{by adding (1) + (2) } \omega &+ (3) \omega^2 \\
 \Rightarrow \text{we get} \\
 3^n &= p_0(1 + \omega + \omega^2) + p_1(1 + \omega^2 + \omega^4) + p_2(1 + \omega^3 + \omega^3) + \dots \\
 3^n &= 3(p_2 + p_5 + p_8 + \dots) \\
 \Rightarrow 3^{n-1} &= p_2 + p_5 + p_8 + \dots \\
 \text{when } n &= 2010 \\
 \Rightarrow p_2 + p_5 + p_8 + \dots &= 3^{2010-1} \\
 &= 3^{2009}
 \end{aligned}$$

87. Answer (3)

We have

$$\begin{aligned}
 b &= \sum_{r=0}^n \frac{r}{\binom{n}{r}^k} = \sum_{r=0}^n \frac{n-r}{\binom{n}{n-r}^k} \\
 &= \sum_{r=0}^n \frac{n}{\binom{n}{r}^k} - \sum_{r=0}^n \frac{r}{\binom{n}{r}^k} = na - b \\
 \Rightarrow 2b &= na \\
 \Rightarrow \cos^{-1}\left(\frac{b}{na}\right) &= \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}
 \end{aligned}$$

88. Answer (4)



From figure

$$f(x) = \begin{cases} 1 - \cos x & 0 \leq x \leq \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \leq x \leq \frac{5\pi}{6} \\ 2 \sin x & \frac{5\pi}{6} \leq x \leq \pi \end{cases}$$

$$\text{so } \int_0^\pi f(x) dx = \int_0^{\frac{\pi}{2}} (1 - \cos x) dx +$$

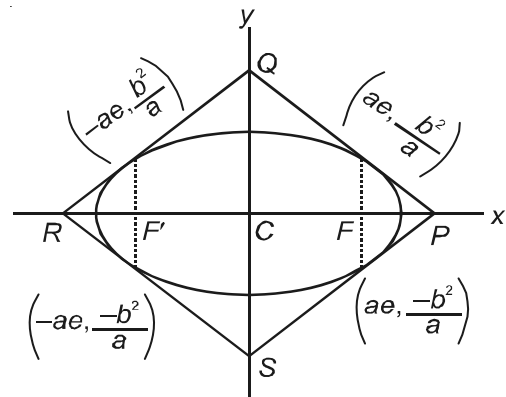
$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} dx + \int_{\frac{5\pi}{6}}^\pi 2 \sin x dx$$

$$\Rightarrow \int_0^\pi f(x) dx = \frac{5\pi}{6} + 1 - \sqrt{3}$$

$$\text{Hence } k = \frac{5\pi}{6}$$

89. Answer (1)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



one end of L.R is  $\left( ae, \frac{b^2}{a} \right)$

equation of tangent at  $\left( ae, \frac{b^2}{a} \right)$  is

$$\frac{xe}{a} + \frac{y}{a} = 1$$

$$\Rightarrow \frac{x}{\frac{a}{e}} + \frac{y}{a} = 1$$

Area of  $\Delta CPQ = \frac{1}{2} \times \frac{a}{e} \times a$

so area of PQRS =  $4 \times \frac{1}{2} \times \frac{a^2}{e}$

$$= \frac{2a^2}{e}$$

90. Answer (4)

$$\text{Max} (|x+y|, |x-y|) = 1$$

If  $|x+y| \geq |x-y|$ , then

$$\max(|x+y|, |x-y|) = |x+y| = 1$$

$\Rightarrow$  If  $4xy \geq 0 \Rightarrow x+y = \pm 1$  which lies in 1st and 3rd quadrants  $x+y = \pm 1$

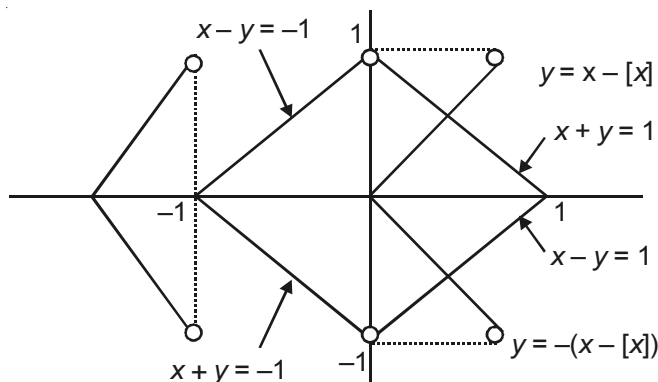
$\Rightarrow$  If  $|x+y| \leq |x-y|$ , then

$$\max(|x+y|, |x-y|) = |x-y| = 1$$

If  $4xy \leq 0$  then  $x-y = \pm 1$  which lies in 2nd and 4th quadrants  $x-y = \pm 1$

As  $|y| = x - [x]$  so  $y = \pm(x - [x])$

as  $n - [n] \geq 0$



for  $-1 \leq x < 0$  two curves coincide  
so there are infinitely many solutions.

