TEST - 13 (Paper - I)

ANSWERS

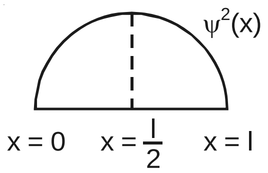
CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(A)	21.	(D)	41.	(D)
2.	(A)	22.	(B)	42.	(A)
3.	(B)	23.	(A)	43.	(C)
4.	(B)	24.	(B)	44.	(B)
5.	(B)	25.	(D)	45.	(A)
6.	(D)	26.	(B)	46.	(C)
7.	(A)	27.	(C)	47.	(B)
8.	(C)	28.	(A)	48.	(B)
9.	(A, C, D)	29.	(A, B, C, D)	49.	(A, C)
10.	(A, B, C)	30.	(A, B, C)	50.	(A, B)
11.	(A, B)	31.	(B, C, D)	51.	(A, C, D)
12.	(A, B)	32.	(A, B, C, D)	52.	(B, C, D)
13.	(B, D)	33.	(C)	53.	(B)
14.	(C)	34.	(A)	54.	(A)
15.	(D)	35.	(D)	55.	(C)
16.	(A)	36.	(A)	56.	(C)
17.	(B)	37.	(B)	57.	(B)
18.	(C)	38.	(C)	58.	(D)
19.	A – (p, q, r, s)	39.	A – (p, q, r, s, t)	59.	A – (p, q)
	B – (p, q, r, s)		B – (q, r, s, t)		B – (q, r, s)
	C – (p, r, s, t)		C – (q)		C – (p, q)
	D – (p, q, r, t)		D – (p)		D – (p, q, r)
20.	A – (r, s, t)	40.	A – (p)	60.	A – (p, q, r, s)
	B – (p, t)		B – (q)		B – (p, r)
	C – (q, r, t)		C – (r)		C – (p, q, s)
	D – (p, r)		D – (q, s, t)		D – (p, r, s)

ANSWERS & HINTS

PART - I (CHEMISTRY)

1. Answer (A)

$$\psi^2(x) = \frac{3}{l}\sin^2\left(\frac{\pi x}{l}\right)$$



Probability of finding electron at the mid-point of line, where $x = \frac{l}{2}$.

$$\psi^2(x) = \frac{3}{l}\sin^2\left(\frac{\pi l}{2\times l}\right) = \frac{3}{l}$$

2. Answer (A)

Due to high partial positive charge

3. Answer (B)

For 1 mole, van der Waal's equation,

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

$$\left(PV + \frac{a}{V}\right)\left(1 - \frac{b}{V}\right) = RT$$

$$\left(PV + \frac{a}{V}\right) = RT\left(1 - \frac{b}{V}\right)^{-1} = RT\left[1 + \frac{b}{V} + \left(\frac{b}{V}\right)^2 + \left(\frac{b}{V}\right)^3 + ...\right]$$

$$PV = RT\left[1 + \frac{\left(b - \frac{a}{RT}\right)}{V} + \frac{b^2}{V^2} + \frac{b^3}{V^3} + ...\right]$$

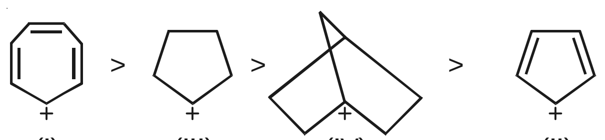
$$P = RT\left[\frac{1}{V_m} + \frac{\left(b - \frac{a}{RT}\right)}{V_m^2} + \frac{b^2}{V_m^3} + \frac{b^4}{V_m^4} + ...\right]$$

On comparing, we get

$$\boxed{B(T) = b - \frac{a}{RT}}$$

4. Answer (B)

Correct sequence of compounds towards S_N1 reactivity



(I) > (III) > (IV) > (II)

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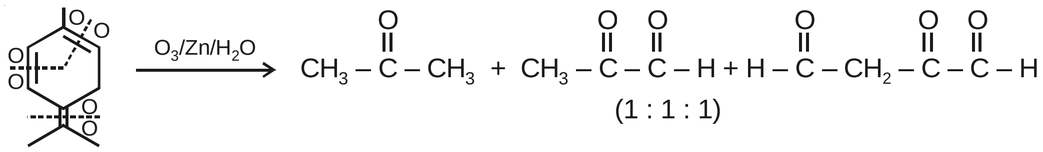
5. Answer (B)



6. Answer (D)

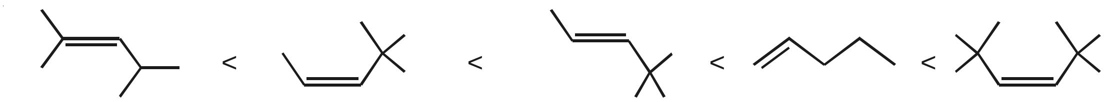
I_3^- contains 3 lone pair electron in central atom.

7. Answer (A)

This is due to the increasing stability of the lower species to which they are reduced.

8. Answer (C)

Greater the stability of alkene lower the heat of hydrogenation.



ΔH = 105 kJ/mole (cis) 114 kJ/mol (trans) 111 kJ/mol 122 kJ/mol 151 kJ/mol

9. Answer (A, C, D)

The buffer capacity increases with increasing ratio of salt : acid. Buffer capacity is maximum at half neutralisation [Salt] = [Acid] reaches to minimum at equivalence point. After equivalence point buffer capacity increases sharply as that is defined as the volume of base of certain concentration added per unit change in pH.

10. Answer (A, B, C)

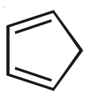
HPh can be used as indicator in the litration of strong base.

11. Answer (A, B)

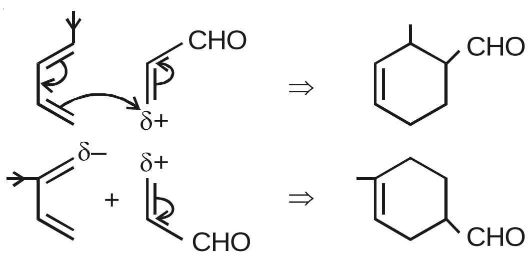
R – F is more reactive than R – Br in Friedel Craft alkylation.

12. Answer (A, B)

13. Answer (B, D)

, it is rigid cis-oid, most favourable.

14. Answer (C)



15. Answer (D)

This is an expample of pericyclic reaction

16. Answer (A)

$$\underset{}{B\ (Boron) + O_2} \longrightarrow \underset{(X)}{B_2O_3}$$

$$\underset{(X)}{B_2O_3} + 6C + 3Cl_2 \longrightarrow \underset{(Y)}{2BCl_3} + 6CO$$

17. Answer (B)

$$\underset{(Y)}{BCl_3} + 3H_2O \longrightarrow \underset{(Z)}{H_3BO_3} + HCl$$

$(\underset{(Z)}{H_3BO_3} + Na_2B_4O_7)$ is used in the preparation of a buffer solution.

18. Answer (C)

$$\underset{(Y)}{4BCl_3} + 3LiAlH_4 \xrightarrow{\text{Ether}} \underset{(T)}{2B_2H_6} + 3LiCl + AlCl_3$$

B_2H_6 is an electron deficient compound.

19. Answer A(p,q, r, s), B(p, q, r, s), C(p, r, s, t), D(p, q, r, t)

20. Answer A(r, s, t), B(p, t), C(q, r, t), D(p, r)

PART - II (MATHEMATICS)

21. Answer (D)

For domain of the given function

$\log_{|[x]|}\{x\} \geq 0$

As $|[x]| \geq 2$, always for domain

$\Rightarrow \{x\} \geq 1$, which is impossible hence number of element = 0.

22. Answer (B)

We have,

$\tan x \sin x - \tan x - \sin x + 1 = 0$ ...(i)

$\Rightarrow \tan x(\sin x - 1) - 1(\sin x - 1) = 0$

$\Rightarrow (\sin x - 1)(\tan x - 1) = 0$

$\Rightarrow \sin x = 1, \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, x \neq \frac{\pi}{2}$

Also,

$\sin^2 x - \sin x \cos x - \sin x + \cos x = 0$

$\Rightarrow \sin x (\sin x - \cos x) - 1(\sin x - \cos x) = 0$

$\Rightarrow (\sin x - \cos x)(\sin x - 1) = 0$

$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, x = \frac{\pi}{2}$

Hence number of common sulutions = 2.

23. Answer (A)

Using the concept of A.M. ≥ H.M.

$$\frac{r_1+r_2+r_3}{3} \ge \frac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{3}{\frac{1}{r}} = 3r$$

$r_1 + r_2 + r_3 \ge 9r = 9 \times 1 = 9$.

Hence minimum value = 9.

24. Answer (B)

Let $f(x) = x^3 - 3x + p$

For exactly 3 distinct roots $f'(x) = 0$

$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x = 1, -1$

For two positive and distinct roots

$f(0) > 0$ and $f(1)\, f(-1) < 0$ must hold together

$\Rightarrow p > 0$ and $(p - 2)(p + 2) < 0 \Rightarrow -2 < p < 2$

$\Rightarrow p \in (0, 2)$

25. Answer (D)

If $\left|z+\frac{1}{z}\right| = a$, then

$$|z|_{max} = \frac{a+\sqrt{a^2+4}}{2}$$

$$|z|_{min} = \frac{-a+\sqrt{a^2+4}}{2}$$

Difference of values = a = 2, according to the question.

26. Answer (B)

The given expression can be written as

$$\left(\sqrt{2}+3^{\frac{1}{3}}+5^{\frac{1}{6}}\right)^{10} = \sum \frac{10!}{x!\,y!\,z!}\, 2^{\frac{x}{2}} 3^{\frac{y}{3}} 5^{\frac{z}{6}}$$

$x + y + z = 10$

There exists three rational terms

for (x, y, z) = (4, 6, 0), (4, 0, 6), (10, 0, 0)

Sum of rational terms = $\frac{10!}{4!6!}2^2 3^2 + \frac{10!}{4!6!}2^2.5 + \frac{10!}{10!}2^5 = 11792$

$\Rightarrow k - 11791 = 1$.

27. Answer (C)

We know that

$S = nA$ ...(i)

$P = G^n$ ...(ii)

$G^2 = AH$ ...(iii)

By (i), (ii), (iii), we get

$$AH = (P)^{\frac{2}{n}} = P^{\frac{2A}{S}}$$

$\Rightarrow \log A + \log H = \dfrac{2A}{S}\log P \Rightarrow k = \dfrac{2A}{S}$.

28. Answer (A)

Let x_1, x_2, x_3 be the number of black, while and red balls.

Required number of ways = non-negative integral solutions of the equation

$x_1 + x_2 + x_3 = 4$

ways = ${}^{4+3-1}C_{3-1} = {}^6C_2 = \dfrac{6\times 5}{2} = 15$

29. Answer (A, B, C, D)

If the point lies inside the circle $x^2 + y^2 = 1$, then point will be outside the hyperbola $x^2 - y^2 = 1$.

Hence probability = 0.

All options are true.

30. Answer (A, B, C)

$$AB = \sqrt{(5-2)^2 + (7-3)^2} = 5$$

Hence options (A), (B) are clearly true.

The smallest circle will be

$(x - 2)(x - 5) + (y - 3)(y - 7) = 0$

$x^2 + y^2 - 7x - 10y + 31 = 0$

31. Answer (B, C, D)

Equation of AB is $2x + 2y - 1 = 0$

$x + y - 1 = 0$

Equation of the required circle H

$(x^2 + y^2 - 1) + \lambda(2x + 2y - 1) = 0$

putting x = 2, y = 2

$(4 + 4 - 1) + \lambda(4 + 4 - 1) = 0$

$$\Rightarrow 7 + 7\lambda = 0, \lambda = -\frac{7}{7} = -1$$

Hence equation is

$$(x^2 + y^2 - 1) - 1(2x + 2y - 1) = 0$$

$$\Rightarrow \quad x^2 + y^2 - 2x - 2y = 0$$

$$\Rightarrow \quad a = b = 1$$

$$c = -2$$

32. Answer (A, B, C, D)

Clearly $a = 1$

(A) Tangents at A and B one perpendicular hence $t_1t_2 = -1$

(B) If chord is normal *at A,* then $t_2 = -t_1 - \frac{2}{t_1}$

(C) If chord is normal at B, then $t_1 = -t_2 - \frac{2}{t_2}$

(D) The directrix of $y^2 = 4x$ is $x = -1$, Hence the angle between the tangents is 90° and in this case, $t_1t_2 = -1$

33. Answer (C)

The given curve may be written as

$$\frac{(x-1)^2}{2} - (y-1)^2 = 1$$

Hence director circle is

$(x - 1)^2 + (y - 1)^2 = 2 - 1 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$

$\Rightarrow$ a = –2, b = –2, c = 1

$a + b + 4c = -2 - 2 + 4 = 0$

34. Answer (A)

The curve C_2 is $(y^2 - x)^2 = 0$

$y^2 = x$

Director circles is directrix that is given by

$$x = -\frac{1}{4} \Rightarrow 4x + 1 = 0$$

$k = 1$

$$\lim_{x\to 1}(1+\sin(x-1))^{\text{cosec }(x-1)} = e^{\lim_{x\to 1}(\sin(x-1).\text{cosec}(x-1))} = e^1 = e$$

35. Answer (D)

If the director of two lines is their point of intersection. Hence area = 0.

36. Answer (A)

$f(x + y) = f(x) + f(y) \Rightarrow f(0) = 0$

at $y = -x$, we find $f(x) + f(-x) = f(0) = 0$

$\Rightarrow f(x) + f(-x) = 0$

$\Rightarrow f(1) + f(-1) = 0$.

37. Answer (B)

As graph is symmetrical about $x = 1$, $x = 2$

$f(1 + x) = f(1 - x)$ ...(i)

$f(2 + x) = f(2 - x)$ ...(ii)

by (i): $x \to x - 1$

$\Rightarrow$ $f(x) = f(2 - x)$ ...(iii)

by (ii): $x \to x - 2$

$f(x) = f(4 - x)$ ...(iv)

by (iii) and (iv)

$f(2 + x) = f(4 + x)$

$\Rightarrow$ $f(x) = f(x + 2)$

$\Rightarrow$ $f(0) = f(2) = f(4) = ...$

but $f(0) = 0$

$\Rightarrow$ $f(2) = f(4) = f(6) = ... = f(8) = 0$

Hence $f(6) + f(4) – f(2) – f(8) = 0$.

38. Answer (C)

As both the points are same hence distance = 0.

39. Answer A(p, q, r, s, t), B(q,r, s, t), C(q), D(p)

(A) $\lim_{x\to 0^+} \frac{[\sin x]}{[x]} = \frac{0}{0}$

(B) $\lim_{x\to 0^+} \left[\frac{e^x - 1}{x}\right] = \lim_{x\to 0^+} \left[\frac{x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...}{x}\right] = 1$

(C) The value of the limit $= e^{\lim_{x\to\infty}\left(\frac{x^2+x+2}{x^2+x+1} - 1\right).(x^2+1)} = e^1 = e$

(D) $\lim_{x\to 0^+} x^x = e^{\lim_{h\to 0} h\log h} = e^{\lim_{h\to 0} \frac{\log h}{\frac{1}{h}}} = e^0 = 1$

40. Answer A(p), B(q), C(r), D(q, s, t)

(A) Projection of A and B on *xy*-plane is $A' \equiv (1, 2, 0)$ and $B' \equiv (4, 6, 0)$. Hence length = $A'B' = 5$

(B) $y = |\sin x|$

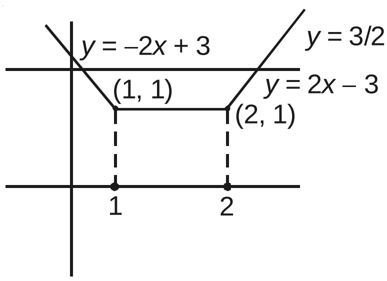
but at $x = \frac{3\pi}{2}$

$y = -\sin x, \frac{dy}{dx} = -\cos x$

at $x = \frac{3\pi}{2}, \frac{dy}{dx} = 0$

(C) $\frac{1}{3} \le x \le 9$

(D) The graph $y = f(x)$ is



Clearly $|x – 1| + |x – 2| = p$ has two solutions in (1, 2010) and one solution when $p = 1$, x = 1, x = 2 and zero solution when $p \in (0, 1)$. The given equation has in infinitely many solution when $p = 1$ and $1 \le x \le 2$

PART - III (PHYSICS)

41. Answer (D)

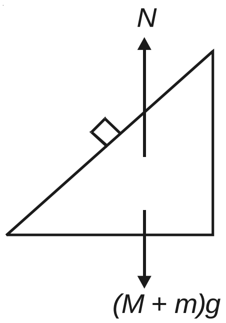
Relative to the balloon, $a = g - a_b = 5$ m/s^2

Distance fallen by first stone = $\frac{1}{2} \times 5 \times 6^2 = 90$ m

Distance fallen by second stone = $\frac{1}{2} \times 5 \times 2^2 = 10$ m

42. Answer (A)

Internal forces acting on the system are as shown with $N < (M + m)g$



43. Answer (C)

$$u = \frac{x^3}{3} - \frac{5x^2}{2} + 6x + 3$$

u is min *at* x = 3

$u_{min} = 7.5$

$k_{max} = 17 - 7.5$

$= 9.5$ J

44. Answer (B)

ω_{point} on circumference = $\frac{\omega_{centre}}{2}$

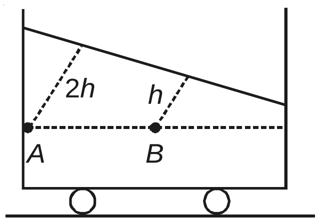
45. Answer (A)

Force on base of vessel = weight of liquid = constant

As area of base increase, pressure on base decreases

46. Answer (C)

47. Answer (B)



$P_B = P_0 + \rho gh$

$P_A = P_0 + 2\rho gh$

$= 2P_B - P_0$

48. Answer (B)

Value of g keeps on decreasing as we move away from earth's surface

49. Answer (A, C)

Frequency of reflected sound = $\frac{w}{\lambda}$

$$= f\left(\frac{w}{w-u}\right)$$

Frequency of sound directly reaching driver of car $f\left(\frac{w+v}{w+u}\right)$

$\Rightarrow$ Beat frequency = $\left|f\left(\frac{w+v}{w+u}\right)-\frac{f(w+v)}{w-u}\right|$

$= \frac{2u(w+v)f}{w^2-u^2}$

50. Answer (A, B)

$F = \frac{mg}{2}, N = \frac{\sqrt{3}mg}{2}$

Resultant of F and N is mg. Angle between resultant of F and N is 120º

51. Answer (A, C, D)

By constraint relation $v_1 = v_2$

$\Rightarrow$ $\omega_1 r_1 = \omega_2 r_2$

kinetic energy of cylinder in pure rolling $= \frac{3}{4}mv^2$

$\Rightarrow$ $m_1 = m_2$ for kinetic energies to be equal angular momentum of any cylinder about

$P = \frac{3mvr}{2} \Rightarrow m_1 r_1 = m_2 r_2$ for angular momentum to be equal

52. Answer (B, C, D)

$H \propto 4y^2$, $\tau \propto 4y$, $r = \frac{u^2 \cos^2\theta}{g}$

53. Answer (B)

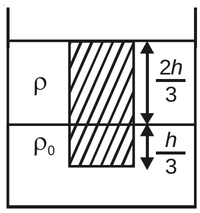
54. Answer (A)

55. Answer (C)

Solution of Q.No. 53 to 55

As the cylinder is half submerged initially

$\Rightarrow$ Density of cylinder = $\frac{\rho_0}{2}$

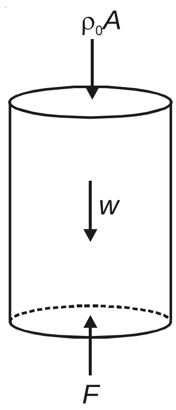


Now, when the lighter liquid has been poured for equilibrium of the cylinder

Weight = Buoyant force

$\Rightarrow \quad \frac{\rho_0}{2}ghA = \rho g\left(\frac{2h}{3}\right)A + \rho_0 g\left(\frac{h}{3}\right)A$

$\rho = \frac{\rho_0}{4}$



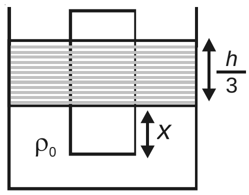
Force at bottom

$F = w + \rho_0 A$

$\Rightarrow \quad F > w$

A several moment is shown as in figure when length of portion of cylinder submerged in lighter fluid is x. Force that needs to be applied = loss in buoyant force

$= \rho_0 g\left(\frac{h}{3} - x\right)\pi r^2$



Force small displacement dx, work done

$dw = \rho_0 g\pi r^2\left(\frac{h}{3} - x\right)dx \quad \Rightarrow \quad w = \rho_0 g\pi r^2 \int_{h/3}^{0}\left(\frac{h}{3} - x\right)dx = \frac{\rho_0 g\pi r^2 h^2}{18}$

56. Answer (C)

57. Answer (B)

58. Answer (D)

Solution of Q.No. 56 to 58

$x = 4\cos^2 at - 1$

$= 2\cos^2 at + 1$

Amplitude = 2 cm

mean position = 1 cm.

$\therefore$ Natural length = 5 cm

Here $\frac{T}{12} = \frac{\pi}{6}$; $T = 2\pi$

min. time is $\frac{T}{4} = \frac{\pi}{2}$ s.

59. Answer : A(p, q), B(q, r, s), C(p, q), D(p, q, r)

AB : Isothermal process, P is increasing

$\Rightarrow$ V is decreasing

$\Rightarrow$ $\Delta U = 0$, $W < 0$, $Q < 0$

BC : Isobaric process, V is decreasing

$\Rightarrow$ $W < 0$, $\Delta U < 0$, $Q < 0$

CD : Isochoric process, V is constant, P is decreasing $\Rightarrow$ T is decreasing

$\Rightarrow$ $W = 0$, $\Delta U < 0$, $Q < 0$

DE : $p^3\rho^{-5}$ = constant $\Rightarrow PV^{\frac{5}{3}}$ = constant

$\Rightarrow$ Process is adiabatic and *V* is increasing

$\Rightarrow Q = 0,\ W > 0,\ \Delta U < 0$

EA : Isobaric process, *V* is increasing

$\Rightarrow W > 0,\ \Delta U > 0,\ Q > 0$

60. Answer : A (p, q, r, s), B(p, r), C(p, q, s), D(p, r, s)

$\vec{p}$ in centre of mass frame is always zero

$\vec{L}$ is conserved if net external torque is zero

$\vec{p}$ is conserved if net external force is zero

kinetic energy is conserved if work done by all the forces sums up to zero mechanical energy is conserved if non-conservative forces don't do any work

TEST - 13 (Paper - II)

ANSWERS

CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(B)	20.	(A)	39.	(B)
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5.	(A, B, D)	24.	(A, B, C)	43.	(B, C, D)
6.	(A, C, D)	25.	(A, B, C)	44.	(C)
7.	(A, C, D)	26.	(A, B, C, D)	45.	(A, C)
8.	(A, B, C)	27.	(A, B, C)	46.	(A, B, D)
9.	(A, B, C)	28.	(A, C, D)	47.	(B, C, D)
10.	A – (p)	29.	A – (p)	48.	A – (p, s)
	B – (p, q, r, t)		B – (q)		B – (r, t)
	C – (p, q, r, t)		C – (p, r, s, t)		C – (p, r, s, t)
	D – (r, s)		D – (p, r, s, t)		D – (p, r, s, t)
11.	A – (p, q)	30.	A – (p, q, r, s, t)	49.	A – (p, r, t)
	B – (p, q, t)		B – (p, r, s, t)		B – (q, r, t)
	C – (s)		C – (p, q)		C – (p, s, t)
	D – (p, r)		D – (p)		D – (q, s, t)
12.	(5)	31.	(1)	50.	(8)
13.	(6)	32.	(2)	51.	(2)
14.	(7)	33.	(4)	52.	(8)
15.	(5)	34.	(6)	53.	(9)
16.	(3)	35.	(1)	54.	(2)
17.	(3)	36.	(1)	55.	(4)
18.	(8)	37.	(3)	56.	(6)
19.	(6)	38.	(2)	57.	(6)

ANSWERS & HINTS

PART - I (CHEMISTRY)

1. Answer (B)

	$CO_{(g)}$ +	$NO_{2(g)}$	$\rightleftharpoons$	$CO_{2(g)}$ +	$NO_{(g)}$
$t = 0$	1 mole	1 mole		0 mole	0 mole
	$1 - x$	$1 - x$		$1 + x$	$1 + x$

$CO_2 + Ba(OH)_2 \longrightarrow BaCO_3$

$$\text{Moles of } BaCO_3 = \frac{236.4}{197} = 1.2$$

So, moles of CO_2 at equilibrium = 1.2

$1 + x = 1.2$

$x = 0.2$

$$\therefore \quad K_C = \left[\frac{1+x}{1-x}\right]^2 = \left(\frac{1.2}{0.8}\right)^2 = 2.25$$

2 Answer (D)

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \Rightarrow \quad \frac{T_2}{T_1} = \frac{6}{3} = 2$$

$$\Delta S = 2.303n\left[C_P \log_{10}\frac{T_2}{T_1} + R\log_{10}\frac{P_1}{P_2}\right]$$

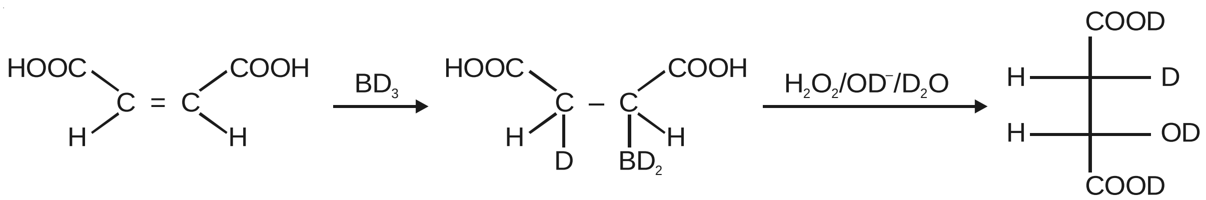
$$= 2.303 \times 1\left[30.96\log_{10}\frac{6}{3} + R\log_{10}\frac{20}{10}\right]$$

$\Delta S = 27.22\ JK^{-1}mol^{-1}$

3 Answer (C)

4. Answer (B)

Hydroboration oxidation is syn hydroxylation (D from BD_3, O from H_2O_2 and proton from solvent)



5. Answer (A, B, D)

But-2-ene is symmetrical alkene, so anti-Markownikoff rule is not applicable

6. Answer (A, C, D)

Conc.H_2SO_4 produces SO_3 electrophile

$2H_2SO_4 \longrightarrow SO_3 + H_3O^+ + HSO_4^-$

7. Answer (A, C, D)

8. Answer (A, B, C)

9. Answer (A, B, C)

Solution of alkali metal in liquid NH_3 is paramagnetic in nature

10. Answer A(p); B(p, q, r, t); C(p, q, r, t); D(r, s)

$$\left(\frac{dT}{dP}\right)_H = \mu$$

$$\frac{dq_{ev}}{T} = \Delta S \qquad \left(\frac{dH}{dT}\right) = C_P$$

$$\left(\frac{dG}{dP}\right) = V$$

11. Answer A(p, q); B(p, q, t); C(s); D(p, r)

Stability of carbanion cannot be explained by hyperconjugation

12. Answer (5)

13. Answer (6)

14. Answer (7)

$$N\% = \frac{1}{8} \times \frac{\text{Volume of } N_2\text{(ml) at S.T.P.}}{\text{wt. of org. compound (gm)}}$$

$$\therefore \quad N\% = \frac{1}{8} \times \frac{25.2}{0.45}$$

= 7%

15. Answer (5)

at T = 298 K

$$\ln K_P = 4.80 - \frac{2033}{298} = 2.02$$

$\therefore$ $\Delta G^o = -RT \ln K_P$

$= -8.31 \times 298 \times (-2.02)$

$= + 5000$ J

$= +5$ kJ

16. Answer (3)

$$CH_3 - CH_2 - \underset{\substack{|\\ I}}{CH_2} \xrightarrow{\text{alc. KOH}} CH_3 - CH = CH_2$$

$\therefore$ Moles of alkene formed = moles of 1-iodopropane × 36%

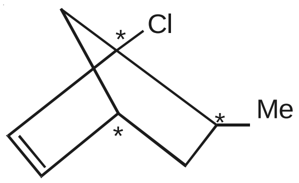
$$= \frac{34}{170} \times 36\%$$

$$\frac{W}{42} = 0.36$$

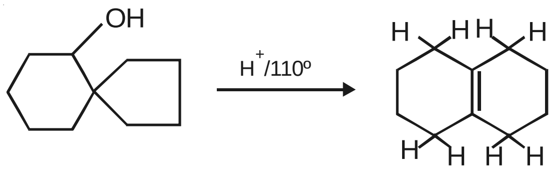
W = 3.02

$W \approx 3g$

17. Answer (3)



18. Answer (8)

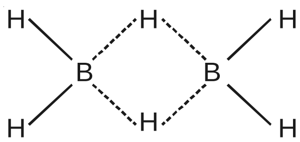


∴ Number of α-hydrogen atoms are 8

19. Answer (6)

$\underset{(x)}{BCl_3} + LiAlH_4 \longrightarrow B_2H_6 \xrightarrow{O_2} B_2O_3$

% of hydrogen in $B_2H_6 = \frac{6}{27.6} \times 100 = 21.72\ \%$



∴ So, number of hydrogen atoms present in same plane are 6

PART - II (MATHEMATICS)

20. Answer (A)

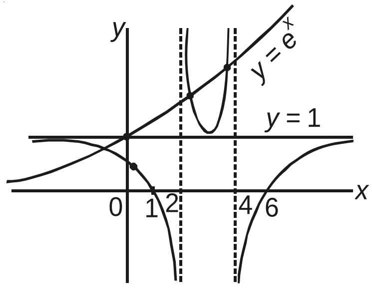
$\log_{|\sin x|}\{x\} \geq 0 \Rightarrow \{x\} \leq 1$

but $\{x\} \neq 0 \Rightarrow x \neq Z$

$|\sin x| \neq 0,1 \Rightarrow x \neq \frac{n\pi}{2}$, hence domain is $R - \frac{n\pi}{2} - Z$

21. Answer (B)

The graph of $y = \frac{x^2 - 7x + 6}{x^2 - 6x + 8} = \frac{(x-1)(x-6)}{(x-2)(x-4)}$ is



Clearly only two solutions exist.

22. Answer (D)

tan 3x – tan 2x – tan x = tan x tan 2x tan 3x

$$\Rightarrow \tan 3x = \frac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$$

$\Rightarrow$ tan 3x = tan(2x + x)

$\Rightarrow$ tan 3x = tan 3x

but $x \neq \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{4}$

23. Answer (D)

Using Napier's analogy

$$\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$$

$$\Rightarrow \tan 15^\circ = \frac{\frac{a}{b}-1}{\frac{a}{b}+1}\cot\frac{C}{2}$$

$$\Rightarrow 2-\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\cot\frac{C}{2} = (2-\sqrt{3})\cot\frac{C}{2}$$

$$\Rightarrow \cot\frac{C}{2} = 1 \Rightarrow C = \frac{\pi}{2}$$

24. Answer (A, B, C)

Let $z = x + iy$, the given equation is

$(x^2 + y^2)(2x^2 - 2y^2) = 160$

$(x^2 + y^2)(x^2 - y^2) = 80$

$\Rightarrow (9 + 1)(9 - 1) = 80$

$\Rightarrow x = \pm 3, y = \pm 1$

Let A = (3, 1), B = (–3, 1), C = (–3, –1), D = (3, –1)

(1) AB = 6, BC = 2

(2) Centre = (0, 0)

(3) Area = 12

(4) Perimeter = 16

25. Answer (A, B, C)

The equation can be written as

$(x - b)(x - c) + (x - a)(x - c) + (x - b)(x - a) = 0$

Let $f(x) = (x - b)(x - c) + (x - a)(x - c) + (x - b)(x - a)$

$\Rightarrow f(a) = (a - b)(a - c)$ = +ve

$f(b) = (b - a)(b - c)$ = –ve

$f(c) = (c - b)(c - a)$ = +ve

Hence, one root lies in (a, b) and other in (b, c).

26. Answer (A, B, C, D)

Let $x - 2 = x_1 \geq 0$

$y - 2 = y_1 \geq 0$

$z - 1 \geq z_1 \geq 0$

$\Rightarrow x = x_1 + 2, y = y_1 + 2, z = 1 + z_1$

By the given equation we find that

$x_1 + 2 + y_1 + 2 + 1 + z_1 = 12$

$x_1 + y_1 + z_1 = 7, x_1, y_1, z_1 \geq 0$

Numbers of solutions = ${}^{7+3-1}C_{3-1} = {}^9C_2 = \dfrac{9 \times 8}{2} = 36.$

27. Answer (A, B, C)

The given equation can be written as

$x!\,(12 - x)! = y!\,(12 - y)!$

$$\Rightarrow \frac{12!}{x!(12-x)!} = \frac{12!}{y!(12-y)!}$$

$\Rightarrow {}^{12}C_x = {}^{12}C_y$

$\Rightarrow x = y$ or $x + y = 12.$

28. Answer (A, C, D)

Let the slope of $L = m$

$$\tan 45° = \pm\frac{m-1}{1+m}$$

$$\Rightarrow 1 = \pm\frac{m-1}{1+m}$$

by (+): $1 = \dfrac{m-1}{1+m} \Rightarrow 1 + m = m - 1 \Rightarrow m \to \infty$

$\Rightarrow L$ is parallel to y-axis

by (–): $1 = -\dfrac{m-1}{1+m} \Rightarrow 1 + m = -m + 1 \Rightarrow m = 0$

Hence, L is parallel to x-axis

Hence, the equation of L is $x = 1$, $y = 1$ and $x + y = 3$

Total area bounded by the lines is $\dfrac{1}{2}$.

29. Answer A(p), B(q), C(p, r, s, t), D(p, r, s, t)

(A) Lines represented are

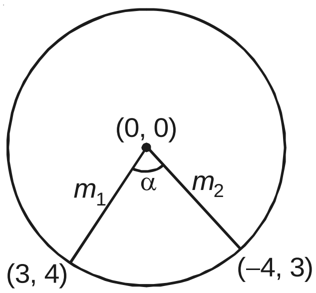
$x + y + 1 = 0$

$x + y + 2 = 0$

Hence, $d = \dfrac{1}{\sqrt{2}} \Rightarrow d\sqrt{2} + 4 = 5.$

(B) As $m_1 m_2 = \left(\frac{4}{3}\right)\left(\frac{3}{-4}\right) = -1$

$\Rightarrow \alpha = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4} \Rightarrow \frac{8\theta}{\pi} = \frac{8}{\pi} \times \frac{\pi}{4} = 2.$



(C) $x^2 + y^2 = \left(\frac{4x + 3y + 5}{\sqrt{25}}\right)^2$

Hence, the focus of parabola is (0, 0) and directrix is $4x + 3y + 5 = 0$

Length of latus rectum = $2\left|\frac{4 \times 0 + 3 \times 0 + 5}{\sqrt{16 + 9}}\right| = 2$

(D) Minimum area = $ab = 2 \times 1 = 2$

30. Answer A(p, q, r, s, t), B(p, r, s, t), C(p, q), D(p)

(A) $-7 \le \sin x + 4\sqrt{3}\cos x \le 7$

(B) If $y = 2x + c$ is tangent to

then $c = \pm\sqrt{2 \times 3 - 2} = \pm\sqrt{4} = \pm 2$

for not tangent, $c \ne \pm 2$

(C) The locus of centre of 'C' will be a hyperbola hence eccentricity is greater than 1.

(D) Equation of chord of contact from $P(\alpha, \beta)$

$\frac{x\alpha}{4} + \frac{y\beta}{3} = 1$ ...(i)

$x + y - 4 = 0$ ...(ii)

By (i) and (ii)

$\frac{\alpha}{4} = \frac{\beta}{3} = \frac{1}{4}$

$\Rightarrow \alpha = 1, \beta = \frac{3}{4}$

$\alpha + \beta = 1 + \frac{3}{4} = \frac{7}{4}$

$\Rightarrow 4(\alpha + \beta) = 7$

31. Answer (1)

$$L = e^{\lim_{x\to 1}\left(\frac{\cos x}{\cos 1}-1\right)\cdot\frac{1}{x-1}}$$

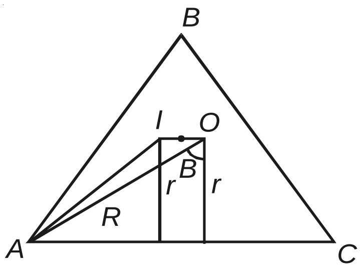
$$= e^{\lim_{x\to 1}\frac{\cos x-\cos 1}{\cos 1}\cdot\frac{1}{x-1}}$$

$$= e^{\lim_{x\to 1}\frac{-\sin x}{\cos 1}} = e^{-\tan 1}$$

$\Rightarrow \log L = -\tan 1$

$$\Rightarrow \left|\frac{\log L}{\tan 1}\right| = 1.$$

32. Answer (2)



I = In centre
O = Circumcentre

$$\cos B = \frac{r}{R} \Rightarrow \cos 60^\circ = \frac{r}{R} \Rightarrow \frac{1}{2} = \frac{r}{R}$$

$$\frac{R}{r} = 2.$$

33. Answer (4)

$$f(x) = x\cos\left(\frac{1}{x}\right)$$

$$f'(x) = -x\sin\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) + \cos\left(\frac{1}{x}\right)$$

$$= \frac{\sin\left(\frac{1}{x}\right)}{x} + \cos\left(\frac{1}{x}\right)$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{\sin\left(\frac{1}{x}\right)}{x} + \cos\left(\frac{1}{x}\right)\right) = 0 + 1 = 1$$

$\Rightarrow k = 1 \Rightarrow k + 3 = 1 + 3 = 4.$

34. Answer (6)

If $n!$ is divisible by 10, then

n = 5, 6, 7, 8, 9, 10

$$p = \frac{6}{10} \Rightarrow 10p = 6.$$

35. Answer (1)

$$d = \sqrt{(|\sin x| + |\cos x|)^2}$$

$$= \sqrt{1 + |\sin 2x|} \Rightarrow 1 \le d \le \sqrt{2}.$$

Hence, minimum value = 1.

36. Answer (1)

$$y = \frac{x^3 + 2x^2 + 2x + 1}{x^2 + x + 1} = \frac{(x+1)(x^2 + x + 1)}{(x^2 + x + 1)} = x + 1$$

$$\Rightarrow \frac{dy}{dx} = 1.$$

37. Answer (3)

$$\lim_{x \to 0} \frac{(1 + e^x) - (\cos x + \sin^2 x) - \cos x.e^x}{x^k}$$

$$= \lim_{x \to 0} \frac{\cos^2 x - \cos x e^x + e^x - \cos x}{x^k}$$

$$= \lim_{x \to 0} \frac{\cos x(\cos x - e^x) - 1(\cos x - e^x)}{x^k}$$

$$= \lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^k}$$

$$= \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}(e^x - \cos x)}{x^k}$$

$$= \lim_{x \to 0} \frac{2\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2 . \left(\frac{x}{2}\right)^2 (e^x - \cos x)}{x^k}$$

$$= \lim_{x \to 0} \frac{1}{2} . \frac{(e^x - \cos x)}{x^{k-2}}$$

$$= \lim_{x \to 0} \frac{1}{2}\left(\frac{\left(1 + x + \frac{x^2}{2!} + ...\right) - \left(1 - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!}...\right)}{x^{k-2}}\right)$$

$$= \lim_{x \to 0} \frac{1}{2}\left[\frac{1 + \frac{2x}{2!} + \frac{x^3}{4!} + ...}{x^{k-3}}\right]$$

For non-zero finite value of limit, $k = 3$.

38. Answer (2)

For infinite solution $a = b = c = 0$

$$\Rightarrow p = \frac{1}{10^3} \Rightarrow 10^3 p + 1 = 2.$$

PART - III (PHYSICS)

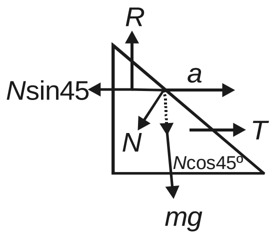
39. Answer (B)

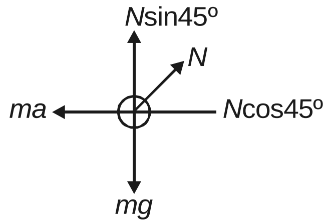
$T - N\sin 45 = ma$ ... (1)

$T\sin 45^\circ = mg$ ... (2)

$N\cos 45^\circ = ma$ ... (3)

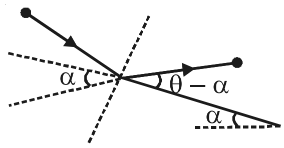
$T = mr\omega^2$ ... (4)





So, $\omega = \sqrt{\dfrac{2g}{r}}$

40. Answer (A)



The figure shows directions of motion of ball before and after the collision

$\Rightarrow \quad \theta - \alpha = \alpha$

$\Rightarrow \quad \theta = 2\alpha$

41. Answer (A)

By impulse momentum theorem

$mv = 2m\, V_{cm} \qquad \Rightarrow \quad V_{cm} = \dfrac{v}{2}$

By angular impulse

$mv\dfrac{\ell}{2} = 2\left(\dfrac{m\ell^2}{2}\right)\omega \qquad \Rightarrow \quad \omega = \dfrac{v}{2}$

Speed of $A = \left|V_{cm} - \omega\dfrac{\ell}{2}\right| = 0$

42. Answer (B)

If constant force F is applied or removed from equilibrium position amplitude of oscillation = $\dfrac{2F}{k}$

43. Answer (B, C, D)

Thermal current = $kA\dfrac{dT}{dx}$ = constant in steady state

44. Answer (C)

$v = \sqrt{\dfrac{rRT}{M}} = \sqrt{\dfrac{rP}{\rho}}$

45. Answer (A, C)

For no slipping $\mu \geq \dfrac{\tan\theta}{1 + \dfrac{mR^2}{I}} \qquad \Rightarrow \quad 1 \geq \dfrac{\tan\theta}{1 + \dfrac{5}{2}}$

$\Rightarrow \quad \tan\theta \leq 2/7$

$\Rightarrow \quad \theta \leq \tan^{-1} 2/7$

$\Rightarrow$ For $\theta < \tan^{-1} 2/7, \qquad k \propto \sin\theta, W_{\text{friction}} = 0$

For $\theta > \tan^{-1} 2/7, \qquad k \propto (\sin\theta - \cos\theta), W_{\text{friction}} < 0$

46. Answer (A, B, D)

Centre of mass of rod A is at geometrical centre as the mass is symmetrically distributed about centre of the rod

$(I_{cm})_{rodA} < (I_{cm})_{rodB}$ as linear mass density near centre is larger

$\Rightarrow$ $(I_{end})_{rodA} < (I_{end})_{rodB}$ by parallel axis theorem

47. Answer (B, C, D)

48. Answer : A(p, s), B(r, t), C(p, r, s, t), D(p, r, s, t)

(A) Tangential acceleration = zero

if $\vec{v}.\dfrac{d\vec{v}}{dt} = \vec{v}.\vec{a} = 0$

(B) Radial acceleration is zero

if $\vec{a} \times \vec{v} = 0$

(C) $\vec{v} = a\sin\omega t\hat{i} + b\cos\omega t\hat{j}$

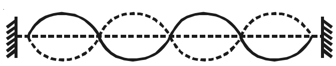
$\Rightarrow$ SHM, UCM, motion on ellipse are possible

(D) $\dfrac{d^2\vec{r}}{dt^2} + k\vec{r} = 0$

$\Rightarrow$ $\vec{r} = \vec{a}\sin\omega t\hat{i} + \vec{b}\cos\omega t\hat{j}$

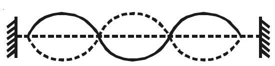
49. Answer : A (p, r, t), B(q, r, t), C(p, s, t), D(q, s, t)

String fixed at both ends



3[rd] overtone

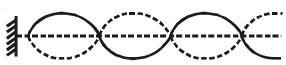
$\ell = 2\lambda = \dfrac{4\pi}{k}$



3[rd] harmonic

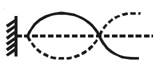
$\ell = \dfrac{3\lambda}{2} = \dfrac{3\pi}{k}$

String free at one end



3[rd] overtone

$\ell = \dfrac{7\lambda}{4} = \dfrac{7\pi}{2k}$



3[rd] harmonic

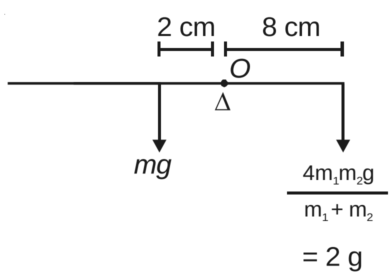
$\ell = \dfrac{3\lambda}{4} = \dfrac{3\pi}{2k}$

50. Answer (8)

For equilibrium about knife edge O

$2g \times 8 = mg \times 2$

$\Rightarrow$ $m = 8$ kg



51. Answer (2)

By $\alpha = -\omega^2\theta$ $T = \sqrt{8/3}$

52. Answer (8)

$u'\cos 45^\circ = u\cos 60^\circ$

$\Rightarrow \quad u' = \frac{u}{\sqrt{2}} \qquad\qquad \Rightarrow \quad k' = \frac{k}{2} = 8\,\text{J}$

53. Answer (9)

Impulse = $\Delta p = 2mv\cos 120^\circ = mv$

$= 3 \times 3 = 9$ kg m/s

54. Answer (2)

$F = \rho a v^2 = 2\rho agh$

$= 2 \times 10^3 \times 10^{-4} \times 10 \times 1$

$= 2$ N

55. Answer (4)

$V^{12}T^4$ = constant $\qquad \Rightarrow TV^3$ = constant

$\Rightarrow PV^4$ = constant

$\Rightarrow \frac{dB}{dP} = 4$

56. Answer (6)

57. Answer (6)

By Newton's law $F = mg\tan\frac{\theta}{2}$

$= 6$ N

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