TEST - 6 (Paper - II)

ANSWERS

CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(C)	21.	(D)	41.	(B)
2.	(A)	22.	(B)	42.	(C)
3.	(A)	23.	(C)	43.	(A)
4.	(C)	24.	(C)	44.	(B)
5.	(A)	25.	(A)	45.	(B)
6.	(C)	26.	(D)	46.	(C)
7.	(B)	27.	(B)	47.	(C)
8.	(D)	28.	(B)	48.	(C)
9.	(B, C)	29.	(A, C)	49.	(A, D)
10.	(A, C, D)	30.	(A, B, C)	50.	(A, D)
11.	(A, B, C, D)	31.	(A, C)	51.	(A, D)
12.	(A, C)	32.	(A, B, C, D)	52.	(A, C)
13.	(3)	33.	(0)	53.	(4)
14.	(5)	34.	(6)	54.	(4)
15.	(2)	35.	(4)	55.	(6)
16.	(5)	36.	(2)	56.	(2)
17.	(5)	37.	(2)	57.	(3)
18.	(6)	38.	(0)	58.	(6)
19.	A $\rightarrow$ (p)	39.	A $\rightarrow$ (p, q)	59.	A $\rightarrow$ (q)
	B $\rightarrow$ (p, q)		B $\rightarrow$ (t)		B $\rightarrow$ (p)
	C $\rightarrow$ (p, q, r)		C $\rightarrow$ (s)		C $\rightarrow$ (t)
	D $\rightarrow$ (s, t)		D $\rightarrow$ (r)		D $\rightarrow$ (r)
20.	A $\rightarrow$ (p, q, s)	40.	A $\rightarrow$ (p, q, r, s, t)	60.	A $\rightarrow$ (s)
	B $\rightarrow$ (q)		B $\rightarrow$ (p, q, r, s)		B $\rightarrow$ (r)
	C $\rightarrow$ (p, r, t)		C $\rightarrow$ (p, r)		C $\rightarrow$ (r)
	D $\rightarrow$ (p, t)		D $\rightarrow$ (q, s, t)		D $\rightarrow$ (r)

ANSWERS & HINTS

PART - I (CHEMISTRY)

1. Answer (C)

   Conjugated alkene is most stable.

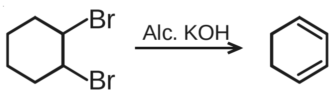
2. Answer (A)

   Structure (A) is 3° allylic carbocation having more resonating structures than structure (D).

3. Answer (A)

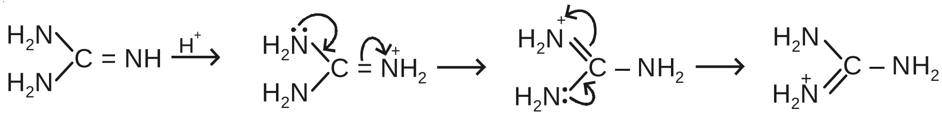
   Closer the –I group to COOH, stronger will be acid and vice-versa.

4. Answer (C)



5. Answer (A)

   Compound (A) is most basic as its conjugate acid is very stable due to resonance.

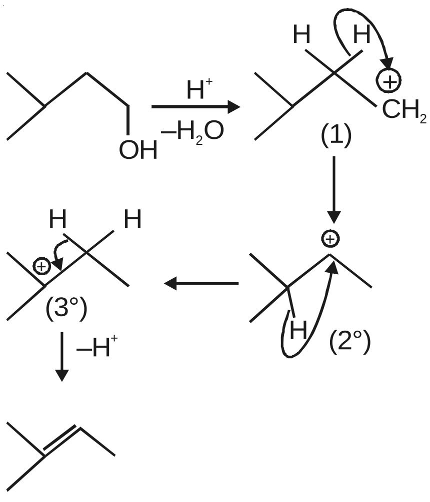


6. Answer (C)

$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH_2\,(g)+6O_2\,(g)\longrightarrow 4CO_2\,(g)+4H_2O\,(l)$$

$$\frac{\text{Volume of } O_2 \text{ consumed}}{\text{Volume of } CO_2 \text{ formed}}=\frac{\text{Number of moles of } O_2}{\text{Number of moles of } CO_2}=\frac{3}{2}$$

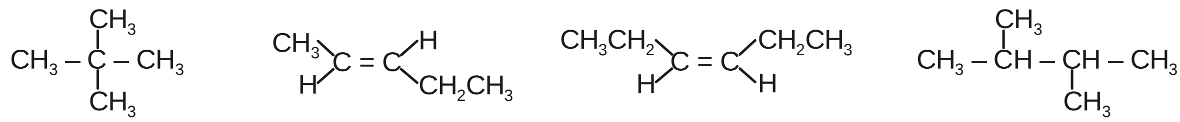
7. Answer (B)



8. Answer (D)

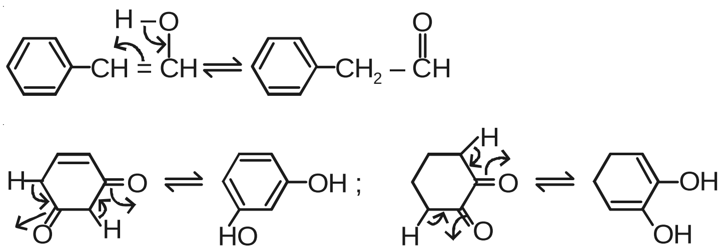
   The given compounds are structural isomers of molecular formula C_5H_{10}. They will form same products on combustion which is exothermic. The magnitude of heat of combustion will depend on stability of reactant. Higher the stability of reactants lower will be the value of heat of combustion. The stability order of the given compounds is (I) < (II) < (III) due to angle strain.

9. Answer (B, C)

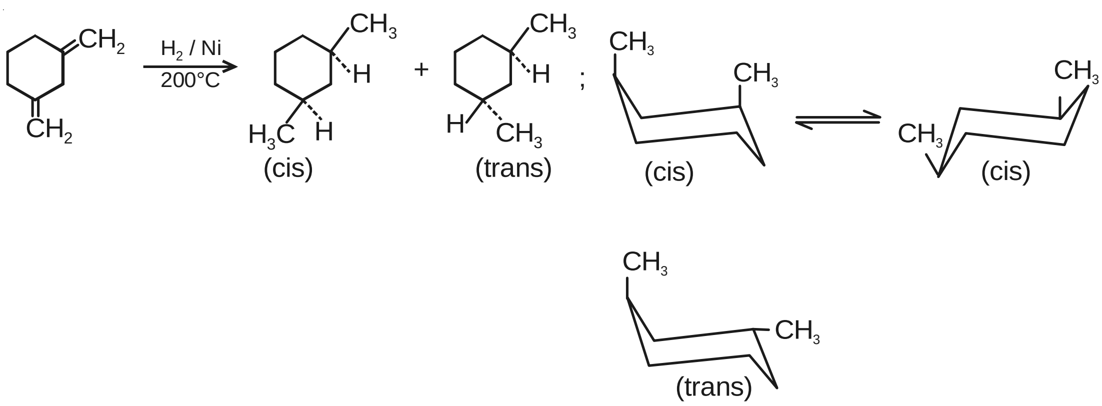


2, 2-dimethylpropane	2-pentene	cis 3-hexene	2, 3-dimethylbutane
(μ = 0)	(μ = +ve)	(μ = +ve)	(μ = 0)

10. Answer (A, C, D)

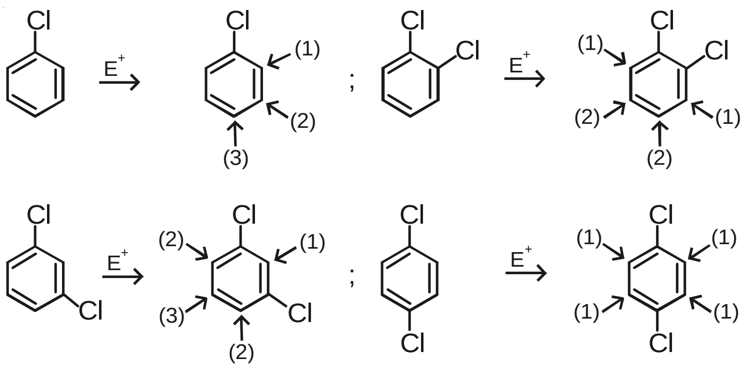


11. Answer (A, B, C, D)



Both the isomers have 6 primary, 8 secondary and 2 tertiary H-atoms.

12. Answer (A, C)

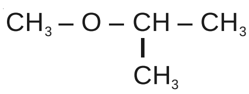


13. Answer (3)

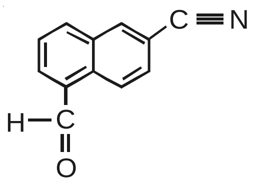
$C_4H_{10}O$ has the following metamers.

$CH_3 – O – CH_2 – CH_2 – CH_3$

$CH_3 – CH_2 – O – CH_2 – CH_3$



14. Answer (5)



$\Rightarrow$ 8π – bonds + 2 ring

x + 5 = 10

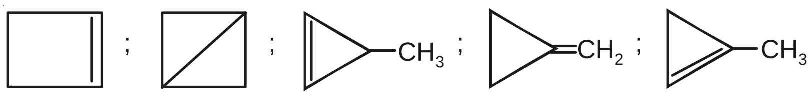
$\Rightarrow$ x = 5

15. Answer (2)

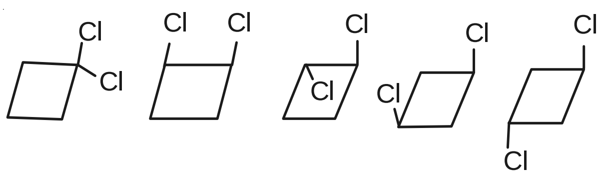
$$\underset{\text{2 moles}}{CH_3CH_2Cl} + Mg \xrightarrow[\text{ether}]{\text{Dry}} CH_3CH_2MgCl \xrightarrow{D_2O} \underset{\text{2 moles}}{CH_3CH_2D\ (g)}$$

16. Answer (5)

Cyclic isomers of C_4H_6 are

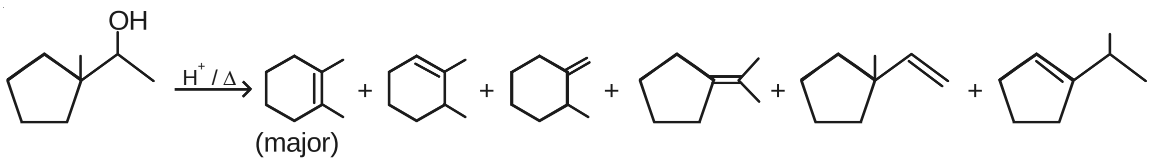


17. Answer (5)



Dichlorination of cyclobutane gives 5 structural and geometrical isomers.

18. Answer (6)



19. Answer A(p), B(p, q), C(p, q, r), D(s, t)

20. Answer A(p, q, s), B(q), C(p, r, t), D(p, t)

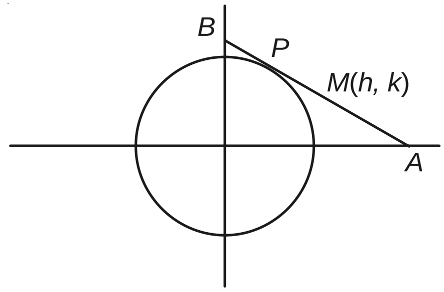
PART - II (MATHEMATICS)

21. Answer (D)

Let a point on the ellipse be $P(4\cos\theta, 3\sin\theta)$. The equation of tangent at P is

$$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{3} = 1$$

It intersects the axes at A and B



$\therefore$ $A\left(\frac{4}{\cos\theta}, 0\right)$ and $B:\left(0, \frac{3}{\sin\theta}\right)$

Let midpoint of AB be $M(h, k)$

$$\therefore \quad h = \frac{\frac{4}{\cos\theta} + 0}{2}, \ k = \frac{\frac{3}{\sin\theta} + 0}{2}$$

$\Rightarrow$ $\cos\theta = \dfrac{2}{h}$

$\sin\theta = \dfrac{3}{2k}$

$\therefore$ $\dfrac{4}{h^2} + \dfrac{9}{4k^2} = 1$

$\therefore$ The locus of (h, k) is $\dfrac{4}{x^2} + \dfrac{9}{4y^2} = 1$

22. Answer (B)

Let a point on hyperbola $\dfrac{x^2}{4} - y^2 = 1$ be $P(2\sec\theta, \tan\theta)$ and the midpoint of the chord of contact drawn from P be $M(h, k)$.

The equation of the chord of contact drawn from P to the ellipse is

$$\frac{x\sec\theta}{2} + y\tan\theta = 1 \quad ...(i)$$

The equation of the chord to ellipse where midpoint is M is, $\dfrac{xh}{4} + yk = \dfrac{h^2}{4} + k^2$ ...(ii)

(i) and (ii) represent the same line

$$\Rightarrow \frac{\sec\theta}{2} \times \frac{4}{h} = \frac{\tan\theta}{k} = \frac{1}{\frac{h^2}{4} + k^2}$$

$$\Rightarrow \sec\theta = \frac{h}{2\left(\frac{h^2}{4} + k^2\right)},\ \tan\theta = \frac{k}{\left(\frac{h^2}{4} + k^2\right)}$$

$\therefore$ $\sec^2\theta - \tan^2\theta = 1$

$$\frac{h^2}{4\left(\frac{h^2}{4} + k^2\right)^2} - \frac{k^2}{\left(\frac{h^2}{4} + k^2\right)^2} = 1$$

$$\Rightarrow \frac{h^2}{4} - k^2 = \left(\frac{h^2}{4} + k^2\right)^2$$

$\therefore$ The locus of (h, k) is

$$\frac{x^2}{4} - y^2 = \left(\frac{x^2}{4} + y^2\right)^2$$

23. Answer (C)

$$\lim_{x \to 0} \frac{x + x^2 + \ln(1 - x)}{x^2}$$

$$= \lim_{x \to 0} \frac{x + x^2 + \left(-x - \frac{x^2}{2} - ....\right)}{x^2} = \frac{1}{2}$$

24. Answer (C)

$$\lim_{x\to 3} \frac{3-\sqrt{f(x)}}{2-\sqrt{x+1}}$$

$$= \lim_{x\to 3} \frac{-\frac{1}{2\sqrt{f(x)}} f'(x)}{-\frac{1}{2\sqrt{x+1}}}$$

$$= \frac{2f'(3)}{\sqrt{f(3)}}$$

$$= \frac{2\times 6}{3} = 4$$

25. Answer (A)

Let $f(x) = ax^2 + bx + c$

where $a < 0$ and $D = b^2 - 4ac < 0$

$f'(x) = 2ax + b$ and $f''(x) = 2a$

$\therefore$ $g(x) = ax^2 + (b + 2a)x + (c + b + 2a)$

$D_1 = (b + 2a)^2 - 4a(2a + b + c)$

$= b^2 + 4a^2 + 4ab - 8a^2 - 4ab - 4ac$

$= -4a^2 + b^2 - 4ac < 0$

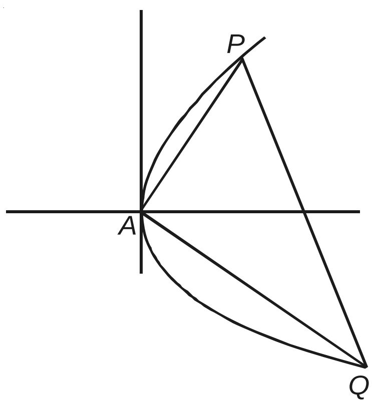
$\therefore$ $g(x) < 0, \ \forall x \in R$

as $D_1, < 0$ and co-efficient of $x^2 = a < 0$

26. Answer (D)

A point on the line $x + 8 = 0$ is $(-8, k)$. The equation of chord of contact is $yk = 4(x - 8)$

$\Rightarrow \dfrac{4x - ky}{32} = 1$



The combined equation AP and AQ (as shown) is $y^2 = 8x\left(\dfrac{4x-ky}{32}\right)$

$\Rightarrow 4x^2 - kxy - 4y^2 = 0$

$\because$ Co-efficient of x^2 + co-efficient of y^2

$= 4 - 4 = 0$

$\Rightarrow$ AP is perpendicular to AQ.

27. Answer (B)

Let D.C.'s of the line be l, m, n

$$\frac{2l}{3}+\frac{1}{3}m+\frac{2}{3}n=0$$

$\Rightarrow$ $2l + m + 2n = 0$ ...(i)

$$\frac{3}{7}l+\frac{6}{7}m+\frac{2n}{7}=0$$

$\Rightarrow$ $3l + 6m + 2n = 0$ ...(ii)

$$\Rightarrow \frac{l}{2-12}=\frac{m}{6-4}=\frac{n}{12-3}$$

$$\Rightarrow \frac{l}{-10}=\frac{m}{2}=\frac{n}{9}$$

$$\Rightarrow \frac{l}{-10}=\frac{m}{2}=\frac{n}{9}=\frac{1}{\sqrt{185}}$$

$$\therefore \left(\frac{-10}{\sqrt{185}}, \frac{2}{\sqrt{185}}, \frac{9}{\sqrt{185}}\right)$$

28. Answer (B)

Let $P(u, v, w)$ be the required point

$\because OP^2 = AP^2 = BP^2 = CP^2$

$\Rightarrow$ $u^2 + v^2 + w^2 = (u - 6)^2 + v^2 + w^2$

$\Rightarrow$ $u = 3$

$u^2 + v^2 + w^2 = u^2 + (v - 8)^2 + w^2$

$\Rightarrow$ $v = 4$

$u^2 + v^2 + w^2 = u^2 + v^2 + (w - 10)^2$

$\Rightarrow$ $w = 5$

$\therefore$ (3, 4, 5) is the required point.

29. Answer (A, C)

$x^2 - 2x - y + 2 = 0$

$(x - 1)^2 = y - 1$

Let the tangent be $(y - 1) = m(x - 1) - \frac{1}{4}m^2$

$\Rightarrow$ $y = mx + 1 - m - \frac{m^2}{4}$ ...(i)

Other parabola is

$-x^2 + 6x - y - 8 = 0$

$x^2 - 6x + 9 = -y - 8 + 9$

$(x - 3)^2 = -(y - 1)$

Equation of tangent with slope m is

$$y-1=m(x-3)+\frac{m^2}{4}$$

$$\Rightarrow \quad y=mx+1-3m+\frac{m^2}{4} \qquad \text{...(ii)}$$

(i) and (ii) represent the same common tangent

$$\Rightarrow \quad 1-m-\frac{m^2}{4}=1-3m+\frac{m^2}{4}$$

$$\Rightarrow \quad \frac{2m^2}{4}=2m$$

$\Rightarrow$ $m = 0$

or $m = 4$

$\therefore$ The equation of the common tangents are $y = 1$ and $y = 4x + 1 - 4 - 4$

i.e. $y = 1$ and $y = 4x - 7$

30. Answer (A, B, C)

If $p \le 2$

$$\lim_{x\to\infty}\frac{x^3+x-1}{x^p+3x^3+1}=\frac{1}{3}$$

If $p = 3$

$$\lim_{x\to\infty}\frac{x^3+x-1}{x^p+3x^3+1}=\frac{1}{4}$$

If $p > 3$

$$\lim_{x\to\infty}\frac{x^3+x-1}{x^p+3x^3+1}=0$$

31. Answer (A, C)

Let the line be $y = 2x + c$

It intersects $xy = 1$

$\Rightarrow$ $x(2x + c) = 1$

$\Rightarrow$ $2x^2 + cx - 1 = 0$

Let the roots be x_1, x_2 (which are in turn the abscissae of A and B)

$$x=\frac{-c\pm\sqrt{c^2+8}}{4}$$

Let $x_1=\dfrac{-c+\sqrt{c^2+8}}{4}$

and $x_2=\dfrac{-c-\sqrt{c^2+8}}{4}$

$\therefore \quad y_1 = 2\left(\dfrac{-c+\sqrt{c^2+8}}{4}\right)+c$

$= \dfrac{c+\sqrt{c^2+8}}{2}$

and $y_2 = \dfrac{c-\sqrt{c^2+8}}{2}$

Let P be (h, k)

$h = \dfrac{3\left(\dfrac{-c-\sqrt{c^2+8}}{4}\right)+\left(\dfrac{-c+\sqrt{c^2+8}}{4}\right)}{4}$

and $k = \dfrac{3\left(\dfrac{c-\sqrt{c^2+8}}{2}\right)+\left(\dfrac{c+\sqrt{c^2+8}}{2}\right)}{4}$

$\Rightarrow \quad 16h = -4c - 2\sqrt{c^2+8}$

$\Rightarrow \quad 8h = -2c - \sqrt{c^2+8}$ ...(i)

$8k = 4c - 2\sqrt{c^2+8}$

$\Rightarrow \quad 4k = 2c - \sqrt{c^2+8}$ ...(ii)

From (i) and (ii)

$8h - 4k = -4c$

$\Rightarrow \quad c = k - 2h$

Put in (i)

$8h + 2(k - 2h) = -\sqrt{c^2+8}$

$(4h + 2k)^2 = (k - 2h)^2 + 8$

$12h^2 + 20hk + 3k^2 = 8$

$\therefore$ The locus of (h, k) is

$12x^2 + 20xy + 3y^2 = 8$

32. Answer (A, B, C, D)

from (A) and (B)

$x^2 + \dfrac{16}{x^2} = 4$

$x^4 - 4x^2 + 16 = 0$

Let x_1, x_2, x_3, x_4 be roots

$\therefore \quad x_1 + x_2 + x_3 + x_4 = 0$

$x_1x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 = -4$

$x_1x_2x_3x_4 = 16,$

Similarly $\quad y_1 + y_2 + y_3 + y_4 = 0$

$y_1y_2y_3y_4 = 16$

33. Answer (0)

The tangent at $A\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ on $\frac{x^2}{9} + \frac{y^2}{4} = \frac{13}{16}$

is $\frac{x\,.\,3\sqrt{3}}{9.2} + \frac{y.1}{4.2} = \frac{13}{16}$

$\Rightarrow \quad \frac{x}{2\sqrt{3}} + \frac{y}{8} = \frac{13}{16}$ ...(i)

Similarly tangent at $B\left(\frac{3\sqrt{3}}{2}, -\frac{1}{2}\right)$ is

$\frac{x}{2\sqrt{3}} - \frac{y}{8} = \frac{13}{16}$ ...(ii)

From (i) and (ii) $P : \left(\frac{13\sqrt{3}}{8}, 0\right)$

Putting P in $x^2 - \frac{y^2}{4} - 1$

$= \frac{169 \times 3}{64} - 0 - 1 > 0$

$\Rightarrow$ P lies inside the hyperbola

$\therefore$ No tangent can be drawn from P to $x^2 - \frac{y^2}{4} = 1$

34. Answer (6)

$l + m - n = 0$

$\Rightarrow \quad n = l + m$

$3l^2 + m^2 - n^2 = 0$

$\Rightarrow \quad 3l^2 + m^2 - (l + m)^2 = 0$

$\Rightarrow \quad 2l^2 - 2lm = 0$

$l = 0, l = m$

$\therefore \quad n = m, n = 2m$

$\Rightarrow \quad \frac{l}{0} = \frac{m}{1} = \frac{n}{1}$ and $\frac{l}{1} = \frac{m}{1} = \frac{n}{2}$

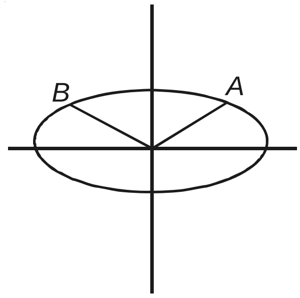
$\therefore \quad \cos\theta = \frac{0 + 1 + 2}{\sqrt{2}\,\sqrt{6}} = \frac{3}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{2}$

$e = \frac{\pi}{6}$

35. Answer (4)

Let A and B are ends of conjugate diameters.



$A : \left(4\sqrt{2}\cos\theta, 3\sqrt{2}\sin\theta\right)$ and

$B : \left(-4\sqrt{2}\sin\theta, 3\sqrt{2}\cos\theta\right)$

$\therefore$ The tangents at A and B are

$$\frac{x\cos\theta}{4\sqrt{2}} + \frac{y\sin\theta}{3\sqrt{2}} = 1 \quad \text{...(i)}$$

and $-\dfrac{x\sin\theta}{4\sqrt{2}} + \dfrac{y\cos\theta}{3\sqrt{2}} = 1 \quad$ ...(ii)

Squaring and adding (i) and (ii)

$$\frac{x^2}{32} + \frac{y^2}{18} = 2$$

$$\Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 4$$

36. Answer (2)

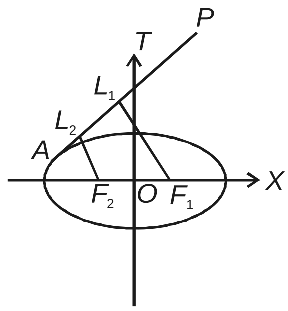
$0 < x < 1$

$\Rightarrow -1 < 3x^3 - 1 < 2$

The number of integers in (– 1, 2) is 2. $\lim\limits_{x \to a} f(x)$ does not exist for the values of a in (0, 1).

37. Answer (2)

Let A be $\left(2\cos\theta, \sqrt{2}\sin\theta\right)$



Equation of the tangent PA is $2x\cos\theta + 2\sqrt{2}y\sin\theta = 4$

$\Rightarrow x\cos\theta + \sqrt{2}y\sin\theta = 2$

$$e = \sqrt{1 - \frac{2}{4}} = \frac{1}{\sqrt{2}}$$

$\therefore$ foci : $\left(\pm\sqrt{2}, 0\right)$

Product of perpendiculars $= \left|\dfrac{\sqrt{2}\cos\theta - 2}{\sqrt{\cos^2\theta + 2\sin^2\theta}}\right| \left|\dfrac{-\sqrt{2}\cos\theta - 2}{\sqrt{\cos^2\theta + 2\sin^2\theta}}\right|$

$$= \left|\frac{4 - 2\cos^2\theta}{\cos^2\theta + 2\left(1 - \cos^2\theta\right)}\right| = 2\left|\frac{2 - \cos^2\theta}{2 - \cos^2\theta}\right| = 2$$

38. Answer (0)

$$f(x) = \frac{(10201)^x}{101 + (10201)^x}$$

$$f(1 - x) = \frac{(10201)^{1-x}}{(101) + (10201)^{1-x}}$$

$$= \frac{10201}{101(10201)^x + 10201}$$

$$= \frac{101}{(10201)^x + 101}$$

$$f(x) + f(1-x) = \frac{(10201)^x + 101}{(10201)^x + 101} = 1$$

$\Rightarrow\ f'(x) - f'(1-x) = 0$

39. Answer A(p, q), B(t), C(s), D(r)

Let a normal to the given parabola be $y = mx - 4m - 2m^3$

$\because$ It passes through (h, k)

$\therefore\ 2m^3 + m(4-h) + k = 0$ ...(i)

The roots of (i) are m_1, m_2, m_3

$\therefore$ co-normal points are $\left(2m_i^2, -4m_i\right), i = 1, 2, 3$

$m_1m_2m_3 = -\frac{k}{2}$ ...(ii)

(p) Two of the normals are perpendicular

$\therefore\ m_1m_2 = -1$

from (ii) $-1 \times m_3 = -\frac{k}{2}$

$\Rightarrow\ m_3 = \frac{k}{2}$, putting in (i)

$2\left(\frac{k}{2}\right)^3 + (4-h)\left(\frac{k}{2}\right) + k = 0$

$k \neq 0$

$k^2 + 8 - 2h + 4 = 0$

$\Rightarrow\ k^2 = 2(h-6)$

$\therefore$ The locus of (h, k) is $y^2 = 2(x-6)$ ...(iii)

(q) $\because$ Feet of two normals are extrimities of focal chord, therefore $m_1m_2 = -1$

again we get locus as (iii)

(r) Let one normal make angle α, then other makes angle $\frac{\pi}{2} - \alpha$

$\therefore\ m_1m_2 = \tan\alpha.\tan\left(\frac{\pi}{2} - \alpha\right) = 1$

Putting in (ii)

$1 \times m_3 = -\frac{k}{2}$

$\Rightarrow m_3 = -\frac{k}{2}$

Putting in (i)

$2\left(-\frac{k}{2}\right)^3 + (4-h)\left(-\frac{k}{2}\right) + k = 0$

$k \neq 0,$

$-k^2 - 8 + 2h + 4 = 0$

$\therefore\ k^2 = 2(h-2)$

$\therefore$ The locus of (h, k) is $y^2 = 2(x-2)$

(s) Directrix of $y^2 = 8x$ is $x + 2 = 0$

Take a point on it as $(-2, y_1)$

Putting in (C) as A, B, D represent parabola

$4 - y_1^2 - 8 + 4 = 0$

$\Rightarrow y_1 = 0$

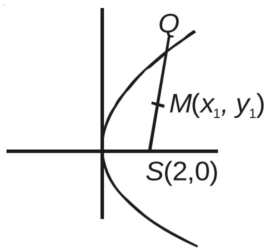
$\therefore$ a point on directrix $(-2, 0)$

The equation of pair of tangents from $(-2, 0)$ to parabola is

$(y^2 - 8x)(16) = (-4(x-2))^2$

$\Rightarrow x^2 - y^2 + 4x + 4 = 0$

(t) Let $Q(2t^2, 4t)$ be a point of $y^2 = 8x$ and is (x_1, y_1) be the midpoint of SQ.



$$x_1 = \frac{2t^2 + 2}{2}, y_1 = \frac{4t + 0}{2}$$

$$\Rightarrow x_1 = \left(\frac{y_1}{2}\right)^2 + 1$$

$$\Rightarrow y_1^2 = 4(x_1 - 1)$$

$\therefore$ The locus of (x_1, y_1) is

$y^2 = 4(x - 1)$

40. Answer A (p, q, r, s, t), B(p, q, r, s), C(p, r), D(q, s, t)

(A) $\lim\limits_{x \to 0} (\cos ax + b \sin x)^{1/x} = \sqrt{e}$

$$\Rightarrow \lim_{x \to 0} e^{\frac{\cos ax + b\sin x - 1}{x}} = e^{1/2}$$

$$\Rightarrow \lim_{x \to 0} e^{\frac{-a\sin ax + b\cos x}{1}} = e^{1/2}$$

$$\Rightarrow b = \frac{1}{2}$$

$\Rightarrow a \in R$

(B) $\because \lim\limits_{x \to 0} x^{\lambda + 1} \sin\frac{1}{2x}$ exists

$$\text{L. H. L} = \lim_{h \to 0} (-h)^{\lambda + 1} \sin\frac{1}{-2h}$$

$$= (-1)^{\lambda} \cdot \frac{h^{\lambda}}{2} \cdot \left(\frac{\sin\frac{1}{2h}}{\frac{1}{2h}}\right)$$

$$\text{R.H.L.} = \lim_{h \to 0} (h)^{\lambda + 1} \cdot \sin\frac{1}{2h}$$

$$= \lim_{h \to 0} \frac{h^{\lambda}}{2} \cdot \frac{\sin\frac{1}{2h}}{\frac{1}{2h}}$$

L.H.L. = R.H.L if both approach to zero

(C) $\lim_{x\to\infty} \frac{1-m^x}{1+m^x} = 1$

if $m \in (0, 1)$

(D) $\lim_{x\to\lambda} \left[x^2+1\right]$ may not exist when $\lambda^2 + 1$ is a positive integer. From options, at $x = 1$

L.H.L. = $\lim_{h\to 0}$ $[(1-h)^2+1] = 1$

R.H.L. = $\lim_{h\to 0}$ $[(1+h)^2+1] = 2$

at $x = 2$,

L.H.L. = $\lim_{h\to 0}\left[(2-h)^2+1\right] = 4$

R.H.L. = $\lim_{h\to 0}\left[(2+h)^2+1\right] = 5$

also at $x = -1$

L.H.L = $\lim_{h\to 0}\left[(-1-h)^2+1\right] = 2$

R.H.L = $\lim_{h\to 0}\left[(-1+h)^2+1\right] = 1$

PART - III (PHYSICS)

41. Answer (B)

$$2P_0\left[\frac{1}{\sqrt{3}}+\sqrt{3}\right] = 5V_0$$

$$\frac{P_0}{V_0} = \frac{5\sqrt{3}}{8}$$

$$V_B = \frac{2P_0}{\sqrt{3}} + V_0$$

$$T_B = \frac{\left(\frac{2P_0}{\sqrt{3}}+V_0\right)3P_0}{nR}$$

$$T_A = \frac{P_0V_0}{nR}$$

42. Answer (C)

$$\Delta P = \frac{1}{0.1\times 10^{-4}}$$

$P_{in} = 2 \times 10^5$ Pa

$10^5 V = nR\ 300$

$2 \times 10^5 V = nRT$

$T = 600$ K

$= 327°C$

43. Answer (A)

$$\frac{\rho_{solid}}{\rho_{liquid}} = \text{fraction submerged}$$

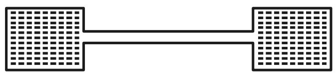
ρ_{solid} is constant therefore $\rho_l f$ = constant

$$\frac{\rho_0}{1+Yt_1}f_1 = \left(\frac{\rho_0}{1+Yt_2}\right)f_2$$

$f_1\ (1 + Y f_2) = f_2(1 + Yt_1)$

$$Y = \frac{f_1 - f_2}{f_2t_1 - f_1t_2}$$

44. Answer (B)



$$n_1 = \frac{p_1V}{RT_1} \qquad n_2 = \frac{p_2V}{RT_2}$$

$$= \left(\frac{pV}{RT}\right) = \left(\frac{p_1}{T_1} + \frac{p_2}{T_2}\right)\frac{V}{R}$$

$$\frac{p}{T} = \frac{1}{2}\left(\frac{p_1}{T_1} + \frac{p_2}{T_2}\right)$$

45. Answer (B)

$$T = T_1 + \frac{T_2 - T_1}{l}x \quad [0 < x < l]$$

$$dt = \frac{dx}{\alpha\sqrt{T}} = \frac{dT}{\alpha\sqrt{T}}\frac{l}{(T_2 - T_1)}$$

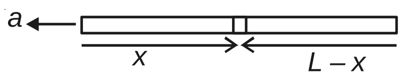
$$t = \frac{2l}{\alpha\,(T_2 - T_1)}\left(\sqrt{T_2} - \sqrt{T_1}\right)$$

$$t = \frac{2l}{\alpha\left(\sqrt{T_2} + \sqrt{T_1}\right)}$$

46. Answer (C)

$$T = \frac{M}{L}(L - x)a$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{(L - x)a}$$



$$\frac{dx}{dt} = \sqrt{(L - x)a}$$

$$t = \int_0^l \frac{dx}{\sqrt{(L - x)a}} = 2\sqrt{\frac{L}{a}}$$

47. Answer (C)

$\beta = 10 \log \frac{I}{I_0}$

$\beta + 5 = 10 \log \frac{I'}{I_0}$

$5 = 10 \log \left(\frac{I'}{I_0}\right)$

$\frac{I'}{I} = 10^{1/2}$

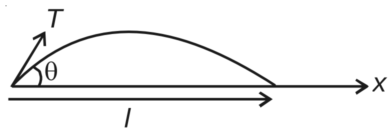
48. Answer (C)

$y = A \cos\omega t \sin kx$

$\frac{\partial y}{\partial x} = Ak \cos \omega t \cos kx$

at $x = 0$

$\frac{\partial y}{\partial x} = Ak \cos \omega t$



$F_y = T \sin\theta \cong T \tan\theta = T \frac{\partial y}{\partial x}$

$F_{y\max} = T \left(\frac{\partial y}{\partial x}\right)_{\max}$

$f = \frac{\sqrt{T/\mu}}{\lambda}$

$\Rightarrow \quad T = (f\lambda)^2 \mu = 10$ N

$F_{y\max} = 2 \times \frac{2\pi}{160} \times 10 = \frac{\pi}{4}$N

49. Answer (A, D)

Temperature gradient = $\frac{dT}{dx}$ = slope of T - x curve

$\left|\frac{dT}{dx}\right| \propto \frac{1}{A}$

50. Answer (A, D)

By increasing the thermal conductivity of outer layer, temperature drop across it decreases. Hence, temperature drop across inner layer increases.

By increasing thickness of inner layer overall heat current decreases hence, temperature drop across outer layer decreases and inner layer increases.

51. Answer (A, D)

(A) $e_A \sigma A_A T_A^4 = \rho_B \sigma A_B T_B^4$

$(0.0256)\,(5800)^4 = (0.0625)\; T_B^4$

$T_B = 4640$ K

(B) $\lambda_A T_A = \lambda_B T_B$

$\lambda_B - \lambda_A = 1\ \mu$m

$\lambda_B = 5\ \mu$m.

52. Answer (A, C)

Intensity ∝ (Amplitude)^2, when medium remaining same.

Also, by conservation of energy, $I_i = I_t + I_r$

53. Answer (4)

$$A_t = \frac{2v_2}{v_1 + v_2}$$

$$A_r = \frac{v_2 - v_1}{v_1 + v_2}$$

$$\frac{A_t}{A_r} = \frac{2v_2}{v_2 - v_1}$$

$$= \frac{2(v_2 / v_1)}{\left(\frac{v_2}{v_1} - 1\right)} = \frac{2 \times 2}{2 - 1} = 4$$

54. Answer (4)

$$f = \frac{mv}{2l} \times 4 = \frac{nv \times 4}{6l}$$

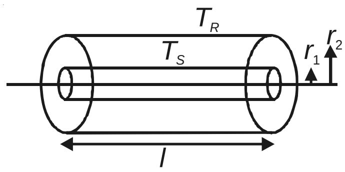
$$\Rightarrow \quad 2m = \frac{2n}{3}$$

$n = 3m$

$f = \frac{2mv}{l}$ For minimum frequency $m = 1$

$$f_{\min} = \frac{2}{l}\sqrt{\frac{T}{\mu}} = \frac{2}{1}\sqrt{\frac{20}{5}} = 4$$

55. Answer (6)



$$\frac{dQ}{dt} = \frac{2\pi k(l)(T_S - T_R)}{\ln(r_2 / r_1)}$$

56. Answer (2)

$$W_1 = nRT_1 \ln\frac{P_A}{P_B}$$

$$W_2 = nRT_2 \ln\frac{P_C}{P_D}$$

$W_1 = 2W_2$

$$T_1 \ln\frac{P_A}{P_B} = 2T_2 \ln\frac{P_c}{P_D}$$

$$\frac{P_A}{T_1} = \frac{P_C}{T_2}$$

$$\frac{P_B}{T_1} = \frac{P_D}{T_2}$$

$$\Rightarrow \frac{P_A}{P_B} = \frac{P_C}{P_D}$$

$$\Rightarrow T_1 = 2T_2$$

$$\frac{T_1}{T_2} = 2$$

57. Answer (3)

PV^m = constant

$$P_m V^{m-1} dv + v^m dp = 0$$

$$\frac{dP}{dV} = -\frac{3}{4} = -\frac{mp}{v}$$

$$m = \frac{3}{4}\frac{V}{P} = \frac{3}{4}\frac{4\times10^5}{2\times10^5} = \frac{3}{2}$$

58. Answer (6)

$$\Delta Q = nC_V\Delta T$$

$$= 2\left[\frac{1\times\frac{3R}{2}+\frac{1\times5R}{2}}{2}\right]\times 900$$

$= 3600\,R$

59. Answer A(q), B(p), C(t), D(r)

$PV = R2T$

$P'V' = RT'$

$P'(2V - V') = 2RT'$

$$\frac{2V - V'}{V'} = 2$$

$$V' = \frac{2V}{3}$$

$u = u_1 + u_2$ = constant

$$1\times\frac{3R}{2}\times 2T + 2\times\frac{5R}{2}\times T = 1\times\frac{3R}{2}T' + 2\times 5\frac{R}{2}\times T'$$

$$8RT = \frac{13R}{2}T'$$

$$T' = \frac{16T}{13}$$

$$\Rightarrow \frac{P'V'}{PV} = \frac{T'}{2T}$$

$\Rightarrow \quad \frac{P'}{P} \times \frac{2}{3} = \frac{16}{2 \times 13}$

$\frac{P'}{P} = \frac{12}{13}$

$= 2 \times \frac{5R}{2}(T' - T)$

$= 5R\left(\frac{3T}{13}\right)$

$= \frac{15}{13}RT$

60. Answer A(s), B(r), C(r), D(r)

$y = a \sin\omega t \sin kx$

$\frac{\partial y}{\partial t} = a\omega \cos \omega t \sin kx$

$\text{KE} = \int_0^{\lambda/2} \frac{1}{2}\frac{m}{l}\, dx\, a^2\omega^2 \sin^2 kx \cos^2 \omega t$

$= \frac{1}{2}\frac{m}{l} a^2\omega^2 \cos^2 \omega t \int_0^{\lambda/2} \sin^2 kx\, dx$

$\Rightarrow l = \frac{\lambda}{2}$

$\text{KE} = \frac{1}{4} m a^2 \omega^2 \cos^2 \omega t$

$\text{KE}_{max} = \text{PE}_{max} = \frac{1}{4} m a^2 \omega^2$

$\langle \text{KE} \rangle = \frac{1}{8} m a^2 \omega^2$

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