TEST - 15

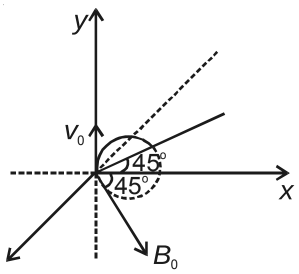
ANSWERS

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	31.	(1)	61.	(1)
2.	(2)	32.	(2)	62.	(4)
3.	(1)	33.	(4)	63.	(4)
4.	(3)	34.	(2)	64.	(4)
5.	(1)	35.	(1)	65.	(4)
6.	(3)	36.	(3)	66.	(4)
7.	(2)	37.	(4)	67.	(1)
8.	(2)	38.	(3)	68.	(4)
9.	(3)	39.	(4)	69.	(4)
10.	(4)	40.	(2)	70.	(1)
11.	(1)	41.	(1)	71.	(2)
12.	(2)	42.	(4)	72.	(3)
13.	(3)	43.	(1)	73.	(4)
14.	(4)	44.	(3)	74.	(1)
15.	(4)	45.	(3)	75.	(4)
16.	(3)	46.	(3)	76.	(4)
17.	(4)	47.	(1)	77.	(1)
18.	(1)	48.	(2)	78.	(4)
19.	(3)	49.	(3)	79.	(1)
20.	(3)	50.	(1)	80.	(4)
21.	(2)	51.	(1)	81.	(2)
22.	(1)	52.	(3)	82.	(1)
23.	(3)	53.	(2)	83.	(2)
24.	(1)	54.	(4)	84.	(2)
25.	(2)	55.	(4)	85.	(3)
26.	(3)	56.	(1)	86.	(2)
27.	(1)	57.	(1)	87.	(3)
28.	(3)	58.	(3)	88.	(4)
29.	(1)	59.	(2)	89.	(1)
30.	(4)	60.	(1)	90.	(4)

Answers & Hints

PART - A : (PHYSICS)

1. Answer (2)



at $t = 0$, $\vec{v}_i = v_0\hat{j}$

at $t = \dfrac{\pi m}{B_0 q}$, $\vec{v}_f = -v_0\hat{j}$

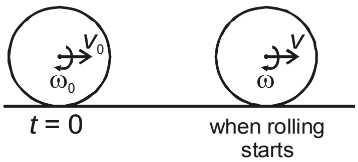
$$\Delta\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$$

$$\Delta\vec{p} = -2mv_0\hat{j}$$

$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} \Rightarrow \vec{F}_{avg} = \left(\frac{-2qB_0v_0}{\pi}\right)\hat{j}$$

2. Answer (2)

$\omega_0 = \dfrac{v_0}{2r}$ $\omega = \dfrac{v}{r}$ $v = \dfrac{6v_0}{7}$

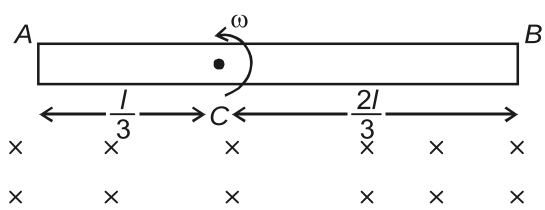


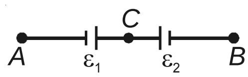
Rotational work done by friction = $\dfrac{1}{2}I\omega^2 - \dfrac{1}{2}I\omega_0^2$

$$= \frac{1}{2}\times\frac{2}{5}mr^2\left(\omega^2 - \omega_0^2\right)$$

$$= \frac{19mv_0^2}{196}$$

3. Answer (1)





$$\varepsilon_1 = \frac{1}{2}B\omega\frac{l^2}{9}$$

$$\varepsilon_1 = \frac{B\omega l^2}{18}$$

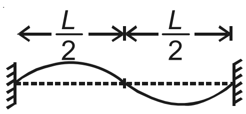
$$\varepsilon_2 = \frac{1}{2}B\omega\times\frac{4l^2}{9}$$

$$\varepsilon_2 = \frac{4B\omega l^2}{18}$$

$$V_A - V_B = -\varepsilon_1 + \varepsilon_2 \Rightarrow V_A - V_B = \frac{B\omega l^2}{6}$$

4. Answer (3) $f = \dfrac{v}{L}$

For n^{th} harmonic $f_n = \dfrac{nv}{2L}$



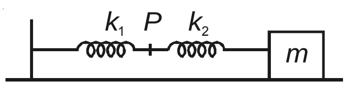
$$\Rightarrow \frac{v}{L} = \frac{nv}{2L} \Rightarrow n = 2$$

Particle at $x = \dfrac{L}{8}$ lies in left loop.

Particle at $x = \dfrac{5L}{8}$ lies in right loop.

So, phase difference between them is π.

5. Answer (1)



Elongation in left spring $x_1 = \dfrac{k_2 x}{k_1 + k_2}$

Where x is total elongation,

So, Amplitude of vibration of P is $\dfrac{k_2 A}{k_1 + k_2}$

6. Answer (3)

PV^2 = constant ____(1)

$B = -V\left(\dfrac{dP}{dV}\right)$ ____(2)

Differentiate eq. (1)

$V^2dP + P \times 2VdV = 0 \Rightarrow 2PVdV = -V^2dP$

$\Rightarrow -V\left(\dfrac{dP}{dV}\right) = 2P \Rightarrow B = 2P$

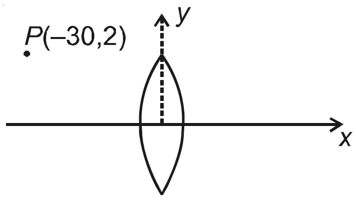
By ideal gas equation

$PV = nRT$ ____(3)

From (1) & (3)

$P \propto T^2 \Rightarrow B \propto T^2$

7. Answer (2)



$f = 10$ cm

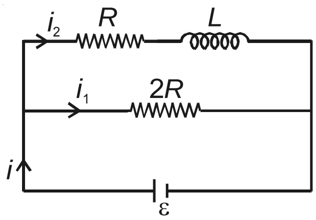
$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = \dfrac{1}{f} + \dfrac{1}{u} \Rightarrow \dfrac{1}{v} = \dfrac{1}{10} + \dfrac{1}{-30}$

$\Rightarrow v = 15$ cm, $x_i = 15$ cm

$\dfrac{y_i}{y_0} = \dfrac{v}{u} \Rightarrow y_i = \dfrac{15}{-30} \times 2 \Rightarrow y_i = -1$ cm

8. Answer (2)

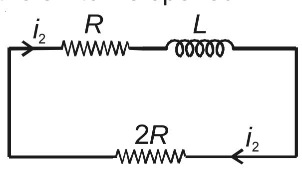
At steady state before opening the switch



$i = i_1 + i_2$

$i_1 = \dfrac{\varepsilon}{2R}$, $i_2 = \dfrac{\varepsilon}{R}$

Just after the switch is opened



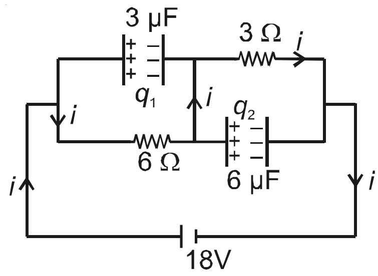
$i_2R + L\dfrac{di_2}{dt} + i_2 \times 2R = 0$

$L\dfrac{di_2}{dt} = -3i_2R$

$L\left(\dfrac{di_2}{dt}\right) = -3\varepsilon \Rightarrow \dfrac{di_2}{dt} = -\dfrac{3\varepsilon}{L}$

9. Answer (3)

$i = 2A$

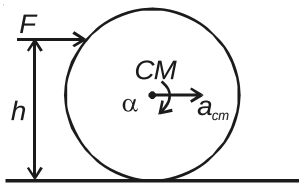


$q_1 = 3 \times 12$

$q_1 = 36$ μC

$q_2 = 6 \times 6 \Rightarrow q_2 = 36$ μC

10. Answer (4)



$a_{cm} = \dfrac{F}{m}$ ____(1)

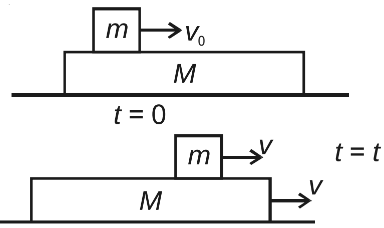
$\alpha = \dfrac{F(h-r)}{I_{cm}}$

$\Rightarrow \alpha = \dfrac{5F(h-r)}{2mr^2}$ ____(2)

For rolling $a_{cm} = r\alpha$

$\Rightarrow \dfrac{F}{m} = \dfrac{5F(h-r)}{2mr} \Rightarrow h = \dfrac{7r}{5}$

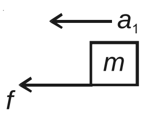
11. Answer (1)



When relative motion stops

By conservation of linear momentum

$mv_0 = (M + m)v \Rightarrow v = \dfrac{mv_0}{M+m}$



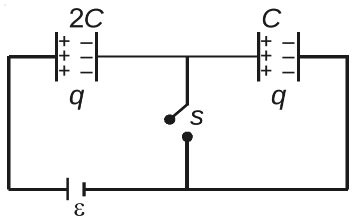
$a_1 = \mu g$ $\quad \bar{v} = \bar{u} + \bar{a}t$

$v = v_0 - \mu gt \Rightarrow \mu gt = v_0 - v$

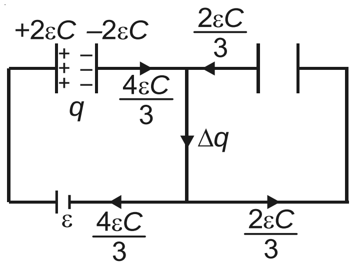
$\Rightarrow \mu gt = v_0 - \dfrac{mv_0}{M+m}$

$\Rightarrow t = \dfrac{mv_0}{(M+m)\mu g}$

12. Answer (2) $q = \varepsilon C_{eq} \Rightarrow q = \dfrac{2\varepsilon C}{3}$



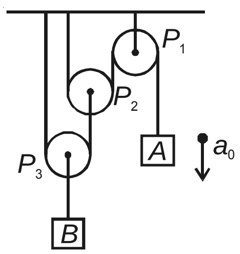
Charge flown through switch



$$\Delta q = \frac{4\varepsilon C}{3} + \frac{2\varepsilon C}{3}$$

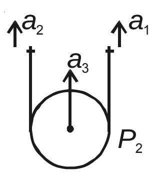
$\Delta q = 2\varepsilon C$

13. Answer (3)



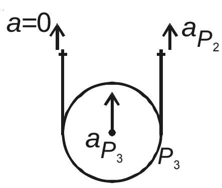
$2a_3 = a_1 + a_2$

$a_3 = a_{P_2}$, $a_1 = a_0$, $a_2 = 0$



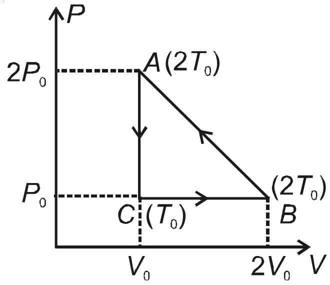
$$a_{P_2} = \frac{a_0}{2}$$

$$a_{P_3} = \frac{a_{P_2}}{2} \Rightarrow a_{P_3} = \frac{a_0}{4}$$



$\Rightarrow \; a_B = \dfrac{a_0}{4}$ upward.

14. Answer (4)



$$T_0 = \frac{P_0V_0}{nR}$$

$$\Delta Q_{BC} = -\frac{5}{2}nRT_0 = -\frac{5}{2}P_0V_0$$

$$\Delta Q_{CA} = \frac{3}{2}nRT_0 = \frac{3}{2}P_0V_0$$

$\Delta Q_{net} = W_{net}$

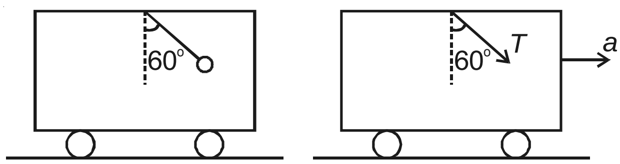
$\Delta Q_{AB} = W_{net} - \Delta Q_{BC} - \Delta Q_{CA}$

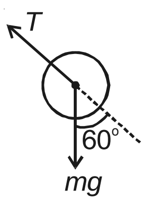
$$\Delta Q_{AB} = \frac{3}{2}P_0V_0$$

$$\eta = \left[1 - \frac{|\Delta Q_{-ve}|}{|\Delta Q_{+ve}|}\right] \times 100 \Rightarrow \eta = \left[1 - \frac{5/2}{3}\right] \times 100$$

$$\Rightarrow \; \eta = \frac{50}{3} \Rightarrow \eta = 16.67\%$$

15. Answer (4)





$T = mg \cos 60°$

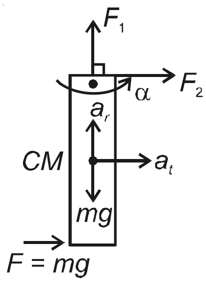
$$\Rightarrow \; T = \frac{1}{2}\,mg$$

$$T \sin 60° = 10m \times a \Rightarrow a = \frac{\sqrt{3}}{4}\ \text{m/s}^2$$

16. Answer (3)

$F \times l = I\alpha$

$\Rightarrow Mg \times l = \frac{1}{3} ml^2\alpha \Rightarrow \alpha = \frac{3g}{l}$



$a_t = \frac{l}{2}\alpha \Rightarrow a_t = \frac{3g}{2}$

$a_r = \frac{l}{2}\omega^2 \Rightarrow a_r = 0$

$F_1 - mg = ma_r \Rightarrow F_1 = mg$

$F + F_2 = ma_t \Rightarrow F_2 = \frac{3}{2}mg - mg \Rightarrow F_2 = \frac{mg}{2}$

Net force by the hinge = $\sqrt{F_1^2 + F_2^2} = \frac{\sqrt{5}}{2}mg$

17. Answer (4)

18. Answer (1)

19. Answer (3)

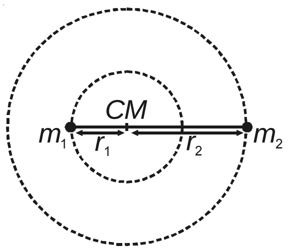
20. Answer (3)

21. Answer (2)

22. Answer (1)

$r_1 + r_2 = r$

$r_1 = \frac{m_2 r}{m_1 + m_2}$ and $r_2 = \frac{m_1 r}{m_1 + m_2}$



$\frac{Gm_1m_2}{r^2} = m_1 r_1 \omega^2$

$\Rightarrow \frac{Gm_1m_2}{r^2} = m_1 \times \left(\frac{m_2 r}{m_1 + m_2}\right)\omega^2$

$\Rightarrow \omega = \sqrt{\frac{G(m_1 + m_2)}{r^3}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{G(m_1 + m_2)}{r^3}}$

$\Rightarrow T = 2\pi\sqrt{\frac{r^3}{G(m_1 + m_2)}}$

23. Answer (3)

$k = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 \Rightarrow k = \frac{1}{2}(I_1 + I_2)\omega^2$

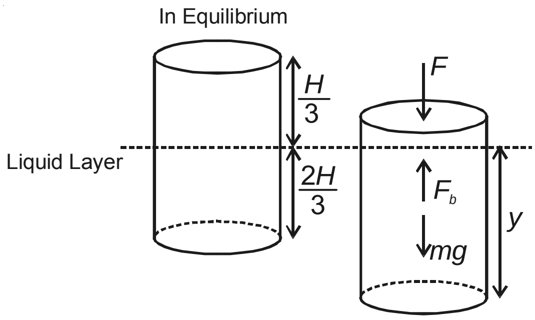
$\Rightarrow k = \frac{1}{2}\left(\frac{m_1m_2}{m_1 + m_2}\right)r^2\omega^2 \Rightarrow k = \frac{Gm_1m_2}{2r}$

24. Answer (1)

$E = K + U$

$= \frac{Gm_1m_2}{2r} + \left(-\frac{Gm_1m_2}{r}\right) \Rightarrow E = -\frac{Gm_1m_2}{2r}$

25. Answer (2)



At equilibrium

$AH\sigma g = A \times \frac{2H}{3} \times \rho g$

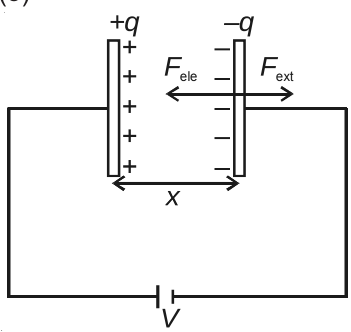
$\Rightarrow \sigma = \frac{2}{3}\rho$

$F + mg = F_b \Rightarrow F = F_b - mg$

$\Rightarrow F = yA\rho g - HA\sigma g \Rightarrow F = Ag\rho\left(y - \frac{2H}{3}\right)$

$dW = Fdy \Rightarrow W = \int_{2H/3}^{H} Fdy \Rightarrow W = \frac{AH^2\rho g}{12}$

26. Answer (3)



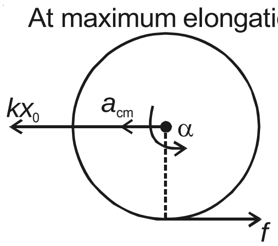
$C = \frac{\varepsilon_0 A}{x}$ $\quad q = \varepsilon V \Rightarrow q = \frac{\varepsilon_0 AV}{x}$

$F_{ele} = \frac{q^2}{2\varepsilon_0 A} \Rightarrow F_{ext} = F_{ele}$

$\Rightarrow F_{ext} = \frac{\varepsilon_0 AV^2}{2x^2} \Rightarrow dW_{ext} = F_{ext}dx$

$W_{ext} = \frac{\varepsilon_0 AV^2}{2}\int_d^{2d}\frac{dx}{x^2} \Rightarrow W_{ext} = \frac{\varepsilon_0 AV^2}{4d}$

27. Answer (1)



$$\frac{1}{2}kx_0^2 = \frac{1}{2}I\omega_0^2 + \frac{1}{2}mv_0^2$$

$$\frac{1}{2}kx_0^2 = \frac{3}{4}mv_0^2 \Rightarrow x_0 = \sqrt{\frac{3m}{2k}}\,v_0$$

$$a_{cm} = \frac{kx_0 - f}{m} \qquad \text{\_\_\_\_(1)}$$

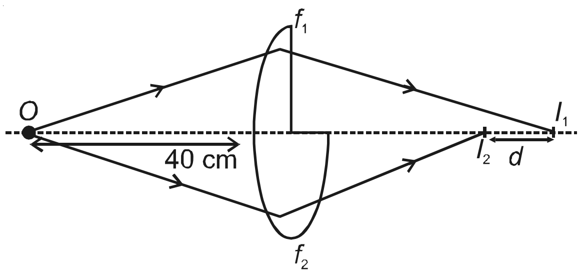
$$\alpha = \frac{2fxr}{mr^2} \Rightarrow \alpha = \frac{2f}{mr} \qquad \text{\_\_\_\_(2)}$$

For rolling $a_{cm} = r\alpha \Rightarrow \frac{2f}{mr} = \frac{kx_0 - f}{m} \Rightarrow f = \frac{kx_0}{3}$

$$\mu N \geq f \Rightarrow \mu mg \geq \frac{kx_0}{3} \Rightarrow \mu mg \geq \sqrt{\frac{mk}{6}}\,v_0$$

$$\Rightarrow \quad \mu \geq \sqrt{\frac{k}{6m}}\frac{v_0}{g} \Rightarrow \mu_{min} = \sqrt{\frac{k}{6m}}\left(\frac{v_0}{g}\right)$$

28. Answer (3)



Focal length of upper half part of lens

$$\frac{1}{f_1} = (1.5 - 1)\left[\frac{1}{10} - \frac{1}{\infty}\right] \Rightarrow f_1 = 20 \text{ cm}$$

Focal length of lower half part of lens

$$\frac{1}{f_2} = (1.5 - 1)\left[\frac{1}{10} - \frac{1}{-10}\right] \Rightarrow \frac{1}{f_2} = \frac{0.5 \times 2}{10}$$

$\Rightarrow$ $f_2 = 10$ cm

Image formation by upper half of lens

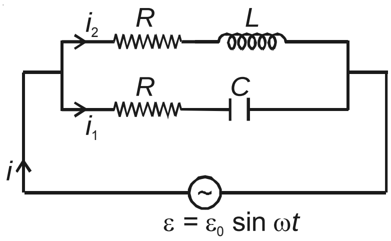
$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \Rightarrow \frac{1}{v_1} = \frac{1}{20} + \frac{1}{-40} \Rightarrow v_1 = 40 \text{ cm}$$

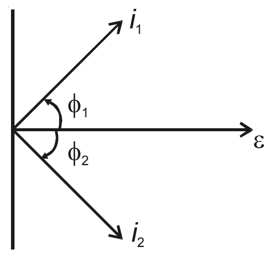
Image formation by lower half of lens

$$\frac{1}{v_2} - \frac{1}{u} = \frac{1}{f_2} \Rightarrow \frac{1}{v_2} = \frac{1}{10} - \frac{1}{-40} \Rightarrow v_2 = \frac{40}{3}$$

$$d = 40 - \frac{40}{3} \Rightarrow d = \frac{80}{3} \text{ cm}$$

29. Answer (1)



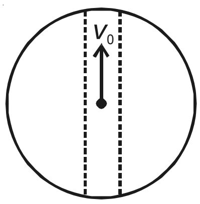


Net current is in phase of e.m.f., if

$$\frac{\varepsilon_0}{z_1}\sin\phi_1 = \frac{\varepsilon_0}{z_2}\sin\phi_2 \Rightarrow \frac{1}{z_1}\times\frac{x_C}{z_1} = \frac{1}{z_2}\times\frac{x_L}{z_2}$$

$$\Rightarrow \quad \frac{x_c}{R^2 + x_c^2} = \frac{x_L}{R^2 + x_L^2} \quad R = \sqrt{\frac{L}{C}}$$

30. Answer (4)



By conservation of mechanical energy

$$\frac{1}{2}mv_0^2 - \frac{3GMm}{2R} = 0 + 0$$

$$\Rightarrow \quad v_0 = \sqrt{\frac{3GM}{R}}$$

PART - B : (CHEMISTRY)

31. Answer (1)

	$PCl_5 \rightleftharpoons$	PCl_3 +	Cl_2
Initially	2	0	0
At eq.	2(1 – x)	2x	2x

$\therefore$ $2(1 - x) + 2x + 2x = 2.5$

$x = 0.25$

32. Answer (2)

$4A \rightarrow 3B + C$

$$-\frac{1}{4}\frac{d[A]}{dt} = \frac{1}{3}\frac{d[B]}{dt} = \frac{d[C]}{dt}$$

$$\frac{1}{4}K_1[A]^4 = \frac{1}{3}K_2[A]^4 = K_3[A]^4$$

$$\frac{K_1}{4} = \frac{K_2}{3} = K_3$$

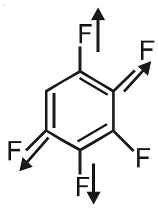
$\therefore 3K_1 = 4K_2 = 12K_3$

33. Answer (4)

34. Answer (2)

$\Delta S > 0$ for irreversible process

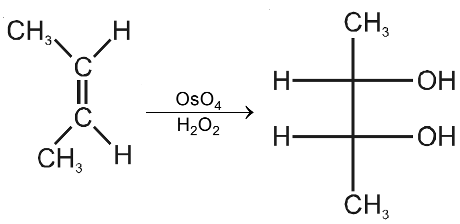
35. Answer (1)



It becomes equal to florobenzene because effect of four flourinc atom cancele to each other.

36. Answer (3)

OsO_4/H_2O_2 gives syn addition



meso form

37. Answer (4)

It is a disproportionation reaction

$3Br_2 + 6NaOH \longrightarrow 5KBr + KBrO_3 + 3H_2O$

38. Answer (3)

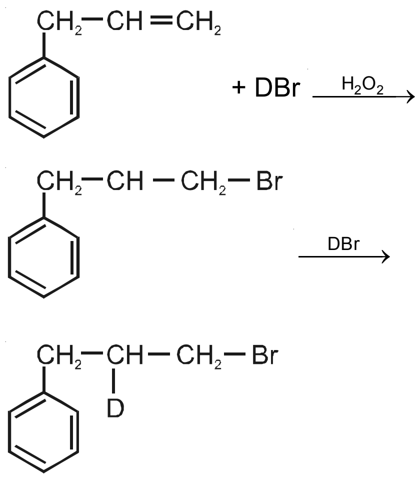
Reduction of $FeCl_3$

$2Na_2S_2O_3 + 2FeCl_3 \rightarrow Na_2S_4O_6 + 2FeCl_2 + 2NaCl$

39. Answer (4)

Number of unpaired electrons are changing

40. Answer (2)



41. Answer (1)

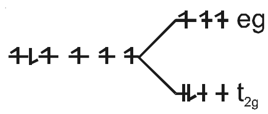
n-factor of NH_3 = 3, $M_1 = 3.E_1$

n-factor of N_2 = 6, $M_2 = 6E_2$

$\therefore M_1 – M_2 = 3E_1 – 6E_2$

42. Answer (4)

In weak field octahedral complex



$\Delta_0 < P$

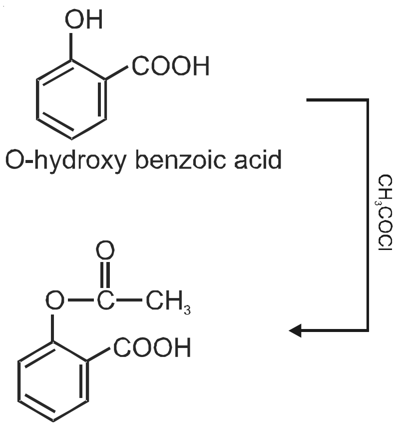
d-orbital

$d^6 \rightarrow t_{2g}^4\ eg^2$

$\therefore$ Difference = 4 – 2 = 2

43. Answer (1)

44. Answer (3)



Aspirin

45. Answer (3)

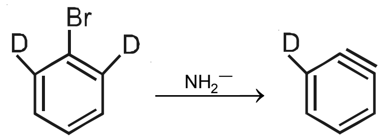
I = Cis-Trans–cis–cis–trans isomer of [10] annulene

II = all cis–isomer of [10] annulene

46. Answer (3)

Cr metal is commercially extracted by Al reduction method.

47. Answer (1)



48. Answer (2)

49. Answer (3)

At high temperature, nucleophilic addition can occur in olefins having -I group attracted to olefinic carbon.

50. Answer (1)

CH_3COOH dimerizes in benzene hence molecular weight is 2 times.

51. Answer (1)

52. Answer (3)

NaCl is the salt of strong acid and strong base. Therefore, no hydrolysis take place and solution will be neutral.

53. Answer (2)

$\because$ 50% NH_3 is neutralised

$NH_4OH + HCl \rightleftharpoons NH_4Cl + H_2O$

$[NH_4^+] = [NH_3]$

$$\therefore \quad p^{OH} = p^{K_b} + \log\frac{[NH_4^+]}{[NH_3]}$$

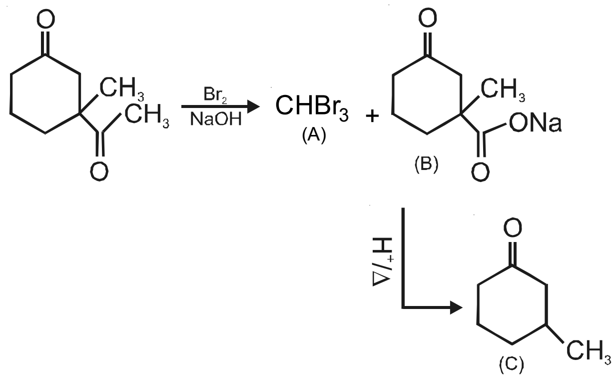
$p^{OH} = 4.72 + \log 1$

$\therefore$ pH = 14 – 4.72

pH = 9.28

54. Answer (4)

55. Answer (4)



56. Answer (1)

57. Answer (1)

$$C_2H_5OH + CH_3COOH \rightleftharpoons CH_3COOC_2H_5 + H_2O$$

At t = 0	1	1	0	0
$t_{eqn.}$	1–x	1 – x	x	x

$\therefore$ x = 2/3

$\Rightarrow$ (1 – 2/3) (1–2/3) 2/3 2/3

$$\therefore K_c = \frac{\frac{2}{3}\times\frac{2}{3}}{\frac{1}{3}\times\frac{1}{3}} = 4$$

$$C_2H_5OH + CH_3COOH \rightleftharpoons CH_3COOC_2H_5 + H_2O$$

t = 0	1	1	0	0
$t_{eqn.}$	(2–x)	2–x	x	x

$$\therefore \quad K_C = \frac{x^2}{(2-x)^2}$$

$$4 = \frac{x^2}{(2-x)^2}$$

$\therefore$ x = 1.33

58. Answer (3)

$$CoCl_3.6H_2O \xrightarrow{AgNO_3\ excess} AgCl$$

0.1 M

Weight of AgCl = n × 0.01 × 143.5

$$n = \frac{2.87}{0.01\times145.5}$$

= 2 (number of AgCl molecules)

$\therefore$ The formula of complex is $[Co(H_2O)_5Cl]\ Cl_2.H_2O$

59. Answer (2)

$$NH_2–SO_3H \xrightarrow[HCl]{NaNO_2} \left[Cl^{\ominus}N_2^{\oplus}–SO_3H\right] \rightarrow H_2SO_4 + NaCl + N_2\uparrow$$

60. Answer (1)

$\Delta G = -\Delta H - T(+\Delta S)$

ΔG = –ve

PART - C : (MATHEMATICS)

61. Answer (1)

It is given that $\sin\theta_1 + \text{cosec}\theta_1 + \tan\theta_2 + \cot\theta_2 = 4$

$\Rightarrow \theta_1 = \frac{\pi}{2}$ and $\theta_2 = \frac{\pi}{4}$

so, $\tan\theta_2 = 1$

$$\Rightarrow 1 = \frac{2\tan\left(\frac{\theta_2}{2}\right)}{1-\tan^2\left(\frac{\theta_2}{2}\right)}$$

$$\Rightarrow \tan^2\left(\frac{\theta_2}{2}\right) + 2\tan\left(\frac{\theta_2}{2}\right) - 1 = 0$$

$\Rightarrow \tan\left(\frac{\theta_2}{2}\right)$ is a root of equation $x^2 + 2x - 1 = 0$

62. Answer (4)

$$I = \int x^2 d\left(e^{2x}\right)$$

$$I = \int x^2 \frac{d}{dx}(e^{2x})dx$$

$$I = 2\int x^2.e^{2x}dx$$

$$I = \frac{1.e^{2x}}{2}\left[2x^2 - 2x + 1\right] + C$$

63. Answer (4)

Let $S = 1 + 2\alpha + 3\alpha^2 + ... + n\alpha^{n-1}$

$\alpha S = \alpha + 2\alpha^2 + ... + n\alpha^n$

$\Rightarrow S(1-\alpha) = 1 + \alpha + \alpha^2 + ... + \alpha^{n-1} - n\alpha^n$

$\Rightarrow S = \frac{1-\alpha^n}{(1-\alpha)^2} - \frac{n\alpha^n}{1-\alpha}$ but $\alpha^n = 1$

so $S = \frac{-n}{1-\alpha}$

64. Answer (4)

The minimum vertical distance between the graphs of given curves

$= \min(2 + \cos x - \sin x)$

$$= \min\left[2+\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)\right]$$

$$= 2-\sqrt{2}$$

65. Answer (4)

Orthocenter of points $\left(a,\frac{1}{a}\right)$, $\left(b,\frac{1}{b}\right)\left(c,\frac{1}{c}\right)$ is given by $\left(-\frac{1}{abc}, -abc\right)$

66. Answer (4)

$\because T_{m+1} = \frac{1}{1-T_m}$

$\Rightarrow T_2 = \frac{1}{1-T_1}$

$T_3 = \frac{1}{1-T_2}$

So, $T_3 = \frac{1}{1-\frac{1}{1-T_1}}$

$T_3 = \frac{1-T_1}{-T_1}$

$T_3 = \frac{T_1-1}{T_1}$ but $T_3 = T_1$

So, $T_1 = \frac{T_1-1}{T_1} \Rightarrow T_1^2 - T_1 + 1 = 0$

Hence, $T_1 = T_3 = T_5 = ...T_{2011} = -\omega$ or $-\omega^2$

then $(-\omega)^{2010}$ or $(-\omega^2)^{2010} = 1$

67. Answer (1)

$\because \left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right] = \left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

and $\left[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}\right] = 2\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

and $2\left[\vec{a}-\vec{b}\ \ \vec{b}-\vec{c}\ \ \vec{c}-\vec{a}\right] = 0$

so given equation can be written as

$32x^2 + 8x = 0$

$\Rightarrow x = 0, -\frac{1}{4}$

68. Answer (4)

Equation of director circle to circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$

69. Answer (4)

Equation of tangent to $y^2 = 8x$ is $y = mx + \frac{2}{m}$

Let (h, k) be the point of intersection

$\Rightarrow k = mh + \frac{2}{m}$

$\Rightarrow m^2h - mk + 2 = 0$

$\Rightarrow m_1 + m_2 = \frac{k}{h}$, $m_1m_2 = \frac{2}{h}$,

$\tan\alpha + \tan\beta = \frac{k}{h}$

$\tan\alpha \tan\beta = \frac{2}{h}$

Given that $\cot\alpha + \cot\beta = 2$

$\Rightarrow \frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta} = c$

$\Rightarrow \frac{\frac{k}{h}}{\frac{2}{h}} = c$

$\Rightarrow k = 2c$

$\Rightarrow y = 2c$

Which is a straight line parallel to x-axis

70. Answer (1)

$\frac{dy}{dx} = \frac{2x - 3y + 7}{3x - 2y + 5}$

$\Rightarrow 3x\,dy + 3y\,dx - 2y\,dy - 2x\,dx + 5dy - 7dx = 0$

$\Rightarrow 3d(xy) - 2y\,dy - 2x\,dx + 5dy - 7dx = 0$

$\Rightarrow 3\int d(xy) - 2\int y\,dy - \int 2x\,dx + 5\int dy - 7\int dx = 0$

$\Rightarrow 3xy - y^2 - x^2 + 5y - 7x + C = 0$

when $x = 0$, $y = 1$

$C = -4$

so, $x^2 + y^2 + 7x - 5y - 3xy + 4 = 0$

71. Answer (2)

Let $y = \frac{mx + p}{n}$, $\bar{y} = \frac{1}{n}(m\bar{x} + p)$

$\Rightarrow y - \bar{y} = \frac{m}{n}(x - \bar{x})$

$\Rightarrow \frac{1}{N}\Sigma(y - \bar{y})^2 = \frac{m^2}{n^2}\frac{1}{N}.\Sigma(x - \bar{x})^2$

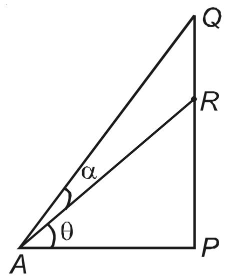
$\therefore$ S.D of $y = \sqrt{\frac{m^2}{n^2}\frac{1}{N}\Sigma(x - \bar{x})^2}$

$= \sqrt{\frac{m^2}{n^2}\sigma^2}$

$\Rightarrow \left|\frac{m}{n}\right|\sigma$

72. Answer (3)

Let $\angle RAP = \theta$



$\tan\theta = \frac{RP}{AP}$

$\tan\theta = \frac{RP}{nPQ}$

$\tan\theta = \frac{2RP}{2nPQ}$

$\tan\theta = \frac{PQ}{2nPQ} \Rightarrow \tan\theta = \frac{1}{2n}$

Now In ΔAQP

$\tan(\alpha + \theta) = \frac{PQ}{AP} = \frac{PQ}{nPQ} = \frac{1}{n}$

so, $\alpha = (\alpha + \theta) - \theta$

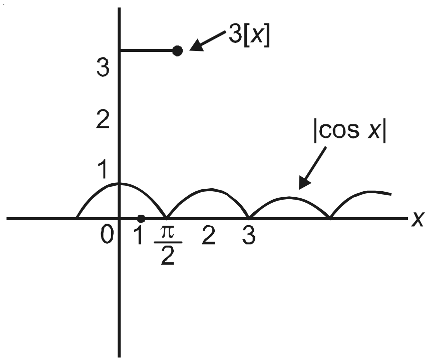
$\Rightarrow \tan\alpha = \frac{\tan(\alpha+\theta) - \tan\theta}{1 + \tan(\alpha+\theta)\tan\theta}$

$\Rightarrow \tan\alpha = \frac{\frac{1}{n} - \frac{1}{2n}}{1 + \frac{1}{n}\frac{1}{2n}}$

$\Rightarrow \tan\alpha = \frac{n}{2n^2 + 1}$

73. Answer (4)

Plot the graph of $|\cos x|$ and $3[x]$



Clearly no solution exists

74. Answer (1)

$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

$\Rightarrow$ $x = y = z = 1$

75. Answer (4)

Equation of line passing through $A(2, -3, -1)$ and $B(8, -1, 2)$ is

$$\frac{x-2}{6} = \frac{y+3}{2} = \frac{z+1}{3} \Rightarrow \frac{x-2}{\frac{6}{7}} = \frac{y+3}{\frac{2}{7}} = \frac{z+1}{\frac{3}{7}}$$

The coordinates of point on this line at a distance 21 from A are

$$\left(2+\frac{6.21}{7}, -3+\frac{2.21}{7}, -1+\frac{3.21}{7}\right)$$

(20, 3, 8)

76. Answer (4)

We have,

$n(S) = 100$

(E) = {even number, square of odd number}

= {2, 4, 6, 8, 10,........., 100, 1, 9, 25, 49, 81}

$n(E) = 55$

Required probability $= \frac{55}{100} = \frac{11}{20}$

77. Answer (1)

78. Answer (4)

79. Answer (1)

80. Answer (4)

81. Answer (2)

Answer of Q.No. 82 to 84

82. Answer (1)

Area of $\Delta APB = 2\times\frac{1}{2}PM\times AM$

but $PM = PA\cos\theta$

so, area = $\sqrt{S_1} . \frac{\sqrt{S_1}}{\sqrt{S_1+r^2}} . \frac{r\sqrt{S_1}}{\sqrt{S_1+r^2}}$

$= \frac{r.S_1^{3/2}}{S_1+r^2}$

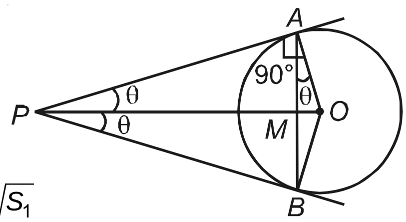
83. Answer (2)

In ΔPAO

$\tan\theta = \frac{OA}{PA} = \frac{r}{\sqrt{S_1}}$

Now $\tan 2\theta = \frac{\frac{2r}{\sqrt{S_1}}}{1-\frac{r^2}{S_1}}$

$\tan 2\theta = \frac{2r\sqrt{S_1}}{S_1-r^2}$



$\Rightarrow 2\theta = \tan^{-1}\frac{2r\sqrt{S_1}}{S_1-r^2}$

Angle between tangents

$= \tan^{-1}\frac{2r\sqrt{S_1}}{S_1-r^2}$

84. Answer (2)

Now $\cos\theta = \frac{AM}{AO}$

$AM = AO\cos\theta$

$AM = r\cos\theta$

$AM = \frac{r\sqrt{S_1}}{\sqrt{S_1+r^2}}$

$AB = 2AM$

$AB = \frac{2r\sqrt{S_1}}{\sqrt{S_1+r^2}}$

85. Answer (3)

We observe that,

$${}^{2010}C_n . {}^{n}C_r = \frac{2010!}{n!(2010-n)!}.\frac{n!}{r!(n-r)!}$$

$$= \frac{2010!}{r!(2010-r)!}\times\frac{(2010-r)!}{(n-r)!(2010-n)!}$$

$= {}^{2010}C_r . {}^{2010-r}C_{n-r}$

Now $\sum_{n=r}^{2010} {}^{2010}C_r . {}^{2010-r}C_{n-r}$

$= {}^{2010}C_r \sum_{n=r}^{2010} \binom{2010-r}{n-r}$

$= {}^{2010}C_r \cdot 2^{2010-r} \because \sum_{r=1}^{n} {}^{n}C_r = 2^n$

Again $\sum_{r=1}^{2010} {}^{2010}C_r \times 2^{2010-r}$

$= 2^{2010} \sum_{r=1}^{2010} \frac{{}^{2010}C_r}{2^r}$

$= 2^{2010} \left[\left(1+\frac{1}{2}\right)^{2010} - 1\right]$

$= 2^{2010} \left[\left(\frac{3}{2}\right)^{2010} - 1\right]$

$= 3^{2010} - 2^{2010}$

86. Answer (2)

$(1 + x + x^2)^n = p_0 + p_1x + p_2x^2 + p_3x^3 + ... + p_{2n} x^{2n}$

Put $x = 1, \omega, \omega^2$ respectively

$3^n = p_0 + p_1 + p_2 + p_3...p_{2n}$ ...(1)

$0 = p_0 + p_1\omega + p_2\omega^2 + p_3 ...p_{2n}\omega^{2n}$ ...(2)

$0 = p_0 + p_1\omega^2 + p_2\omega^4$ ...(3)

$\because 1 + \omega + \omega^2 = 0$

by adding (1) + (2) ω + (3) ω^2

$\Rightarrow$ we get

$3^n = p_0(1 + \omega + \omega^2) + p_1 (1 + \omega^2 + \omega^4) + p_2 (1 + \omega^3 + \omega^3) + ...$

$3^n = 3(p_2 + p_5 + p_8 + ...)$

$\Rightarrow 3^{n-1} = p_2 + p_5 + p_8 + ...$

when $n = 2010$

$\Rightarrow p_2 + p_5 + p_8 + ... = 3^{2010-1}$

$= 3^{2009}$

87. Answer (3)

We have

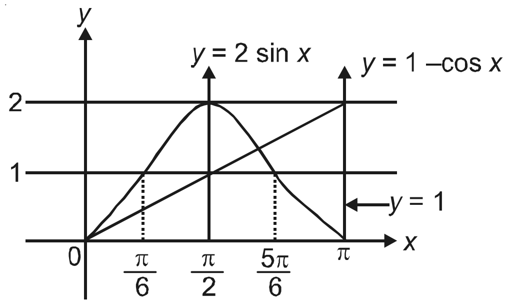
$$b = \sum_{r=0}^{n} \frac{r}{({}^nC_r)^k} = \sum_{r=0}^{n} \frac{n-r}{({}^nC_{n-r})^k}$$

$$= \sum_{r=0}^{n} \frac{n}{({}^nC_r)^k} - \sum_{r=0}^{n} \frac{r}{({}^nC_r)^k} = na - b$$

$\Rightarrow 2b = na$

$\Rightarrow \cos^{-1}\left(\frac{b}{na}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

88. Answer (4)



From figure

$$f(x) = \begin{cases} 1-\cos x & 0 \le x \le \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \le x \le \frac{5\pi}{6} \\ 2\sin x & \frac{5\pi}{6} \le x \le \pi \end{cases}$$

so $\int_0^{\pi} f(x)dx = \int_0^{\frac{\pi}{2}} (1-\cos x)dx +$

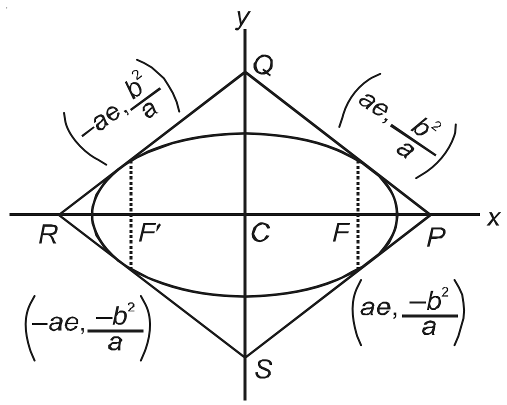
$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} dx + \int_{\frac{5\pi}{2}}^{\pi} 2\sin x\, dx$

$\Rightarrow \int_0^{\pi} f(x)dx = \frac{5\pi}{6} + 1 - \sqrt{3}$

Hence $k = \frac{5\pi}{6}$

89. Answer (1)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



one end of L.R is $\left(ae, \frac{b^2}{a}\right)$

equation of tangent at $\left(ae, \frac{b^2}{a}\right)$ is

$$\frac{xe}{a} + \frac{y}{a} = 1$$

$$\Rightarrow \frac{x}{\frac{a}{e}} + \frac{y}{a} = 1$$

Area of Δ *CPQ* = $\frac{1}{2} \times \frac{a}{e} \times a$

so area of *PQRS* = $4 \times \frac{1}{2}\frac{a^2}{e}$

$= \frac{2.a^2}{e}$

90. Answer (4)

Max $(|x+y|, |x-y|) = 1$

If $|x + y| \geq |x - y|$, then

$\max(|x + y|, |x - y|) = |x + y| = 1$

$\Rightarrow$ It $4xy \geq 0 \Rightarrow x + y = \pm 1$ which lies in 1st and 3rd quadrants $x + y = \pm 1$

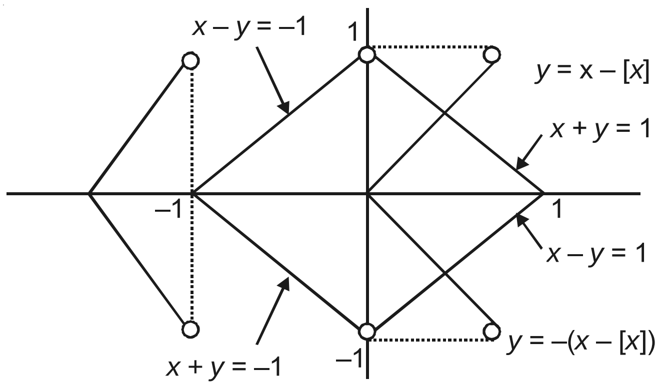
$\Rightarrow$ If $|x + y| \leq |x - y|$, then

$\max(|x + y|), |x - y| = |x - y| = 1$

If $4xy \leq 0$ then $x - y = \pm 1$ which lies in 2nd and 4th quadrants $x - y = \pm 1$

As $|y| = x - [x]$ so $y = \pm(x - [x])$

as $n - [n] \geq 0$



for $-1 \leq x < 0$ two curves coincide

so there are infinitely many solutions.

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