

**TEST - 6****ANSWERS****PHYSICS**

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30. (4)

**CHEMISTRY**

31. (3)
32. (4)
33. (4)
34. (3)
35. (4)
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**MATHEMATICS**

61. (2)
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## PART - A (PHYSICS)

1. Answer (3)

$$KE = \frac{3}{2} RT$$

$$= \frac{3}{2} PV$$

At  $V = V_0$ ,

$$P = \frac{P_0}{1 - \frac{V_0}{V_0}} \frac{P_0}{2}$$

$$= \frac{3}{2} \frac{P_0}{2} V_0 = \frac{3}{4} P_0 V_0$$

2. Answer (2)

$$\Delta u = nC_v \Delta T$$

$$= 2 \cdot \frac{3R}{2} (2T_0 - T_0)$$

$$= 3RT_0$$

$$= \frac{3}{2} P_0 V_0$$

3. Answer (4)

$$l \propto L^2$$

$$l = KL^2$$

$$l + \Delta l = K[L(1 + \alpha\Delta T)]^2$$

$$l + \Delta l = KL^2(1 + 2\alpha \cdot \Delta T)$$

$$\Delta l = 2l\alpha\Delta T$$

$$\frac{\Delta l}{l} = 2\alpha \Delta T$$

4. Answer (1)

$$\frac{l_2}{1 - Yt_2} = \frac{l_1}{1 - Yt_1}$$

5. Answer (4)

$$Y_1 A_1 \frac{(L_1 - 1) T}{L_1} = Y_2 A_2 \frac{L_2 - 2 T}{L_2}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{Y_2 - 2}{Y_1 - 1}$$

6. Answer (1)

$$T = 2\sqrt{\frac{L}{g}}$$

$$T = 2\sqrt{\frac{L(1 - T)}{g}}$$

$$T = T = 2\sqrt{\frac{L}{g}} - \frac{T}{2}$$

$$\text{Time loss per second} = \frac{T}{T} - \frac{T}{2}$$

$$\text{Times loss in one day} = \frac{1}{2} T \cdot 60 \cdot 24 \cdot 60$$

7. Answer (3)

Average translational KE is independent of molecular mass.

8. Answer (2)

$$n_1 = \frac{P_0 V_0}{RT_0}, n_2 = \frac{P_0 V_0}{RT_0}$$

$$n_1 = n_2 = \frac{2P_0 V_0}{RT_0} = \frac{PV_0}{R^2 T_0}$$

$$\Rightarrow 2P_0 = P \cdot \frac{3}{2}$$

$$\Rightarrow P = \frac{4P_0}{3}$$

9. Answer (1)

10. Answer (4)

$$\theta = nC_p(30) \quad \dots(i)$$

$$\theta = nC_v \Delta T \quad \dots(ii)$$

$$\frac{C_p}{C_v} = \frac{30}{T} = 1$$

$$\Delta T = Y \times 30$$

$$= \frac{7}{5} \cdot 30$$

$$\Delta T = 42 \text{ K}$$

11. Answer (3)

$$f_0 = \frac{V}{V} \frac{V_0}{V_0} = 2f_0$$

$$V_0 = \frac{V}{3}$$

12. Answer (2)

$$Y = 0.05 \sin(100\pi t)$$

$$Y = 0.05 \sin(600\pi t - 250\pi x)$$

$$\Delta\phi = 350\pi - 100\pi = 250\pi$$

13. Answer (1)

$$T = 2 \times 10^{-3}$$

After half-time period, phase difference equal  $\pi$ .

14. Answer (3)

$$Y = A \sin(\omega t - kx)$$

$$Y = A \cos(\omega t - kx)$$

$$k(\Delta x) = \pi$$

$$x = \frac{\pi}{k} = \frac{\lambda}{2}$$

$$x = \frac{\lambda}{2}$$

15. Answer (4)

$$Y = A \sin(\omega t - kx)$$

$$V = A \omega \cos(\omega t - kx)$$

$$= A \sin \left( \omega t - kx \right) \frac{\omega}{2}$$

$$a = -A\omega^2 \sin(\omega t - kx)$$

$$a = -\omega^2 Y$$

$$\frac{a}{Y} = -\omega^2$$

16. Answer (3)

$$T = AY\alpha t$$

$$V = \sqrt{\frac{AY}{m}} \sqrt{\frac{t}{L}}$$

17. Answer (1)

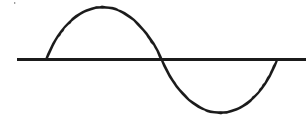
$$T \propto x$$

$$V \propto \sqrt{T} \propto \sqrt{x}$$

$$V \propto \sqrt{\frac{1.5}{1}} \propto \sqrt{1.5} V$$

18. Answer (4)

$$\frac{v}{2} = \frac{v}{4} + \frac{L}{3}$$



19. Answer (1)

$$\sqrt{\frac{vg}{4}} = f \frac{L}{4} \quad \dots(i)$$

$$\sqrt{\frac{vg}{5}} = f \frac{L}{5} \quad \dots(ii)$$

20. Answer (1)

$$\sqrt{\frac{vg}{f_1}} = f_1 \quad \dots(i)$$

$$\sqrt{\frac{vg}{f_2}} = f_2 \quad \dots(ii)$$

21. Answer (1)

$$\log_{10} 2 = 0.3$$

$$\Delta\beta = 10 \log_{10} 5$$

22. Answer (3)

$$f_1 = \frac{v}{2L_1}$$

$$f_2 = \frac{v}{4L_2}$$

$$L_2 = \frac{L_1}{2}$$

23. Answer (3)

$$f_B = \frac{v}{2} \frac{1}{l} = \frac{1}{l} \frac{v}{x}$$

24. Answer (1)

$$\frac{v}{2} \frac{1}{l} = \frac{v}{2} \frac{1}{l} n$$

$$\Rightarrow f_0 = \frac{v}{2l}$$

25. Answer (4)

On increasing the tension beats increases, therefore, frequency of sonometer string is greater.

26. Answer (1)

$$\Delta Q = \Delta u + \Delta w$$

In sudden expansion,  $\Delta Q = 0$ 

$$\Rightarrow 0 = \Delta u + \Delta w$$

$$\Delta u = -\Delta w$$

In expansion,  $\Delta w$  is positive, this  $\Delta u$  is negative and hence the gas cools.

27. Answer (3)

In conduction, bulk motion does not take place.

28. Answer (4)

Kinetic energy of a gas molecule is  $\frac{1}{2}mv^2$ .

29. Answer (1)

$$\lambda T = \text{constant}$$

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$\frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1} = \frac{480}{360}$$

$$\frac{T_1}{T_2} = \frac{4}{3}$$

30. Answer (4)

In closed path, change in internal energy is zero but work done is non-zero.

## PART - B (CHEMISTRY)

31. Answer (3)

Hyperconjugation is feasible for the species having one or more  $\alpha$ -H atoms only.

32. Answer (4)

Heat of hydrogenation of

$$\text{alkene} \propto \frac{1}{\text{stability of alkene}}$$

33. Answer (4)

Positively charged carbon atom in a carbocation is  $sp^2$  hybridised and one of the  $2p$ -orbitals is empty.

34. Answer (3)

Lone pair of electrons on  $-\ddot{\text{O}}\text{CH}_3$  group and  $\pi$  bonded electrons are in conjugation.

35. Answer (4)

Statement is self-explaining.

36. Answer (1)

Higher the number of branching, lower will be surface area, lower will be boiling point.

37. Answer (3)

It is electrophilic substitution which continues till all Cl atoms are substituted.

38. Answer (2)

 $-\text{OCH}_3$  is ring activating group while  $-\text{NO}_2$  is ring deactivating group, hence further nitration will take place in the same ring which has  $-\text{OCH}_3$  group that too at least hindered position, *i.e.*, ortho position.

39. Answer (1)

 $-\text{NH}_3^+$ ,  $-\text{OH}$  are  $-I$  groups $-\text{D}$ ,  $-\text{COO}^-$  are  $+I$  groups

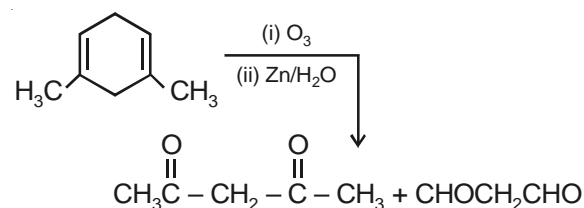
40. Answer (4)

In boat form and chair form of cyclohexane, bond angle (C - C - C) is  $109^\circ 28'$ .

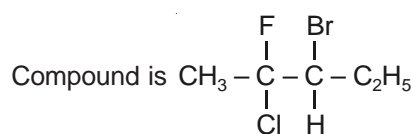
41. Answer (1)

Carbene formation is involved during reaction mechanism.

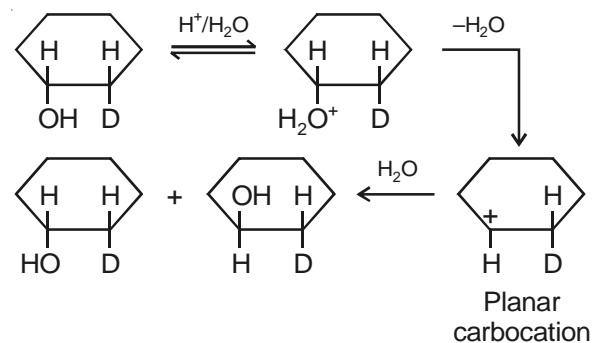
42. Answer (3)



43. Answer (3)



44. Answer (4)



45. Answer (2)

Stability of alkenes is described by inductive effect and total number of hyperconjugative structures.

46. Answer (1)

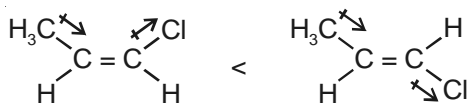
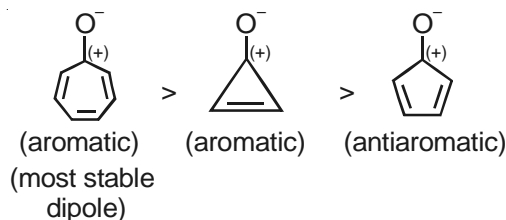
Degree of unsaturation = 0

 $\Rightarrow$  No double bond or ring.

47. Answer (4)

 It is most stable  $\sigma$  complex.

48. Answer (3)


 $-\text{NH}_2$ ,  $-\text{CH}_3$  are  $e^-$  donating while  $-\text{NO}_2$  is  $e^-$  withdrawing group.

49. Answer (3)

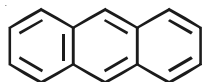
Corey House synthesis is preparation of unsymmetrical alkanes by using dialkylcuprous lithium.

50. Answer (4)

 Order of abstraction of different H atoms is  $3^\circ > 2^\circ > 1^\circ$ .

51. Answer (4)

52. Answer (2)


 $\Rightarrow$  3 rings + 7  $\pi$ -bonds.

53. Answer (2)

 Stability of carbocations follows the order  $3^\circ > 2^\circ > 1^\circ$ , but presence of positive charge on a bridgehead carbon makes it highly and exceptionally unstable.

54. Answer (1)

Conjugated alkene is more stable

55. Answer (2)

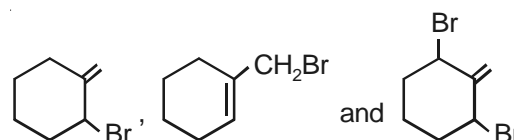
Order of nucleophilicity is same in water as well as in acetone.

56. Answer (4)

 Assertion is wrong as  $\text{CH}_2(\text{Br})-\text{CH}_2-\text{COOH}$  will be the major product and reason given is correct in itself, due to strong  $-I$  effect of  $-\text{COOH}$  group.

57. Answer (3)

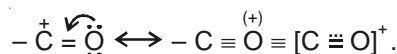
Assertion is correct, because possible products in this reaction may be



but reason is false.

58. Answer (1)

Resonance in acylium ion makes it more stable electrophile.



59. Answer (2)

Fact.

60. Answer (2)

Stagger form is most stable conformer because two methyl groups are away from each other, hence torsional and steric strain is minimized.

## PART - C (MATHEMATICS)

61. Answer (2)

$$\bar{x} = \frac{3 \cdot 4 \cdot 8}{5} = 6$$

$$\Rightarrow \lambda + \mu = 15 \quad \dots(i)$$

$$\sigma^2 = \frac{(x_i - \bar{x})^2}{n}$$

$$= \frac{9 \cdot 4 \cdot 4 \cdot (6)^2 \cdot (6)^2}{5}$$

$$= 6.8$$

$$\Rightarrow (\lambda - 6)^2 + (\mu - 6)^2 = 17$$

$$\Rightarrow \lambda^2 - 12\lambda + 36 + (15 - \lambda - 6)^2 = 17$$

$$\Rightarrow \lambda^2 - 12\lambda + \lambda^2 - 18\lambda + 81 + 19 = 0$$

$$\Rightarrow 2\lambda^2 - 30\lambda + 100 = 0$$

$$\Rightarrow \lambda^2 - 15\lambda + 50 = 0$$

$$\Rightarrow \lambda = 5, 10$$

$$\Rightarrow \lambda = 10, 5$$

62. Answer (1)

$$\lim_{x \rightarrow 1} \frac{\sin(x^3 - 1)}{\sqrt{x} - 1} = \lim_{x \rightarrow 1} \frac{\cos(x^3 - 1) \cdot 3x^2}{\frac{1}{2\sqrt{x}}} = 0$$

63. Answer (3)

$$\sin x = e^y$$

$$\cos x = e^y \frac{dy}{dx}$$

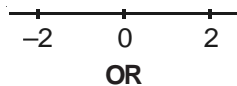
$$\frac{dy}{dx} = \cot x$$

64. Answer (2)

p	q	~q	p ^ q	p ^ q -> p	(p ^ ~q)	q ^ ~q	~(q ^ ~q)
T	T	F	T	T	F	F	T
T	F	T	F	T	T	F	T
F	T	F	F	T	F	F	T
F	F	T	T	T	F	F	T

65. Answer (1)

No such x exists minimum value of (|x + 2| + |2 - x|) is 4.



(i)  $x < -2$

$$-(x + 2) - (x - 2) < 3$$

$$\Rightarrow -2x - 3 < x - \frac{3}{2}$$

$\Rightarrow$  No value of x

(ii)  $-2 \leq x \leq 2$

$$(x + 2) - (x - 2) < 3$$

$$4 < 3$$

$\Rightarrow$  No value of x

(iii)  $x > 2$

$$(x + 2) + (x - 2) < 3$$

$$\Rightarrow 2x < 3 \Rightarrow x < \frac{3}{2}$$

$\Rightarrow$  No value of x

66. Answer (4)

$$f(x) = (1 - x)^{200} \Rightarrow f(0) = 1$$

$$f'(x) = -200(1 - x)^{199} \Rightarrow f'(0) = -200$$

$$f''(x) = 200 \cdot 199(1 - x)^{198} \Rightarrow f''(0) = 200 \cdot 199$$

$\vdots$

$$f^{(r)}(x) = (-1)^r 200 \cdot 199 \dots (200 - r + 1) \cdot (1 - x)^{200-r}$$

$$\Rightarrow f^{(200)}(x) = 200 \cdot 199 \dots (201 - r)$$

$$\therefore f(0) = \frac{f(0)}{1!} - \frac{f'(0)}{2!} + \dots + \frac{f^{(200)}(0)}{200!}$$

$$= 1 - \frac{200}{1!} + \frac{200 \cdot 199}{2!} - \dots$$

$$+ \dots + (-1)^r \frac{200 \cdot 199 \dots (200 - r + 1)}{r!}$$

$$+ \dots + (-1)^{200} \frac{200!}{200!}$$

$$= {}^{200}C_0 - {}^{200}C_1 + {}^{200}C_2 - \dots + {}^{200}C_{200}$$

$$= 0$$

67. Answer (1)

$$x = e^{y+x}$$

Differentiating w.r.t. x

$$1 = e^{x+y}(1 + y')$$

$$1 + y' = e^{-(x+y)} \Rightarrow y' = e^{-(x+y)} - 1$$

68. Answer (3)

$$\lim_{x \rightarrow \infty} \frac{x^3 \sqrt{1 + \frac{x^2}{x^6} + \frac{1}{x^6}}}{x^3 - 2 + \frac{1}{x} - \frac{1}{x^3}} = \frac{1}{2}$$

69. Answer (4)

Let the mean be  $\bar{x}$ .

$$\sum_{i=1}^n (x_i - 6i)$$

$$\therefore \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{x} = \frac{3n(n+1)}{n}$$

$$\bar{x} = 3(n+1)$$

70. Answer (3)

The perpendicular distance of (2, -3, 4) from the x-axis

$$= \text{Distance between } (2, -3, 4) \text{ and } (2, 0, 0)$$

$$= \sqrt{0^2 + 9 + 16}$$

$$= 5$$

71. Answer (1)

$$\text{LHL} = \lim_{h \rightarrow 0} [11 - 11h] - [11h - 11]$$

$$+ [11 - h] + [-11 + h]$$

$$= 0 - 1 + 10 - 11$$

$$= -2$$

$$\begin{aligned} \text{RHL} &= \lim_{h \rightarrow 0} [11 \ 11 \ h] [11 \ h \ 11] \\ &\quad + [11 + h] + [-11 - h] \\ &= -1 + 0 + 11 - 12 \\ &= -2 \end{aligned}$$

$$\therefore \lim_{x \rightarrow 11} ([11 \ x] [x \ 11] [x] [x]) = 2$$

72. Answer (2)

$$\log_{1/5}(x^2 + 2x + 2) + 1 > 0$$

$$0 < (x^2 + 2x + 2) \cdot \frac{1}{5} < 1$$

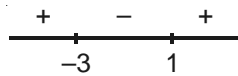
$$\therefore x^2 + 2x + 2 > 0, \forall x \in R$$

$$\therefore x^2 + 2x + 2 < 5$$

$$x^2 + 2x - 3 < 0$$

$$(x + 3)(x - 1) < 0$$

$$\therefore x \in (-3, 1)$$



73. Answer (1)

Let direction cosines of the required line be  $l, m, n$ .

$$\Rightarrow \frac{12l}{13} = \frac{4m}{7} = \frac{3n}{3}$$

$$\Rightarrow 7(12l + 4m - 3n) = 13(6l + 3m - 2n)$$

$$\Rightarrow 6l - 11m + 5n = 0 \quad \dots(i)$$

$$\Rightarrow 3(6l + 3m - 2n) = 7(2l + 2m - n)$$

$$\Rightarrow 4l - 5m + n = 0 \quad \dots(ii)$$

From (i) and (ii),

$$\frac{l}{11} = \frac{m}{20} = \frac{n}{30}$$

$$\Rightarrow \frac{l}{14} = \frac{m}{14} = \frac{n}{14}$$

$$\Rightarrow \frac{l}{1} = \frac{m}{1} = \frac{n}{1} = \frac{1}{\sqrt{3}}$$

$$\therefore \text{Direction cosines are } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

74. Answer (4)

$$\lim_{x \rightarrow 1} \frac{ax^3 - 1}{1 - x^2} = b$$

$$\therefore \text{Limit exists finitely} \Rightarrow a - 1 = 0, \text{ i.e., } a = 1$$

$$\lim_{x \rightarrow 1} \frac{x - 1}{1 - x^2} = b$$

$$\Rightarrow \lim_{x \rightarrow 1} \frac{\frac{1}{x} - \frac{1}{x^2}}{\frac{1}{x^2} - 1} = b$$

$$\Rightarrow b = -1$$

75. Answer (2)

$$\bar{x} = \frac{1 \cdot d + 1 \cdot 3d + \dots + 1 \cdot 99d}{50}$$

$$= 1 + d \cdot \frac{50 \cdot 100}{50}$$

$$= 1 + 50d$$

$\therefore$  Mean deviation ( $\bar{x}$ )

$$= \frac{1}{50} (|(1 - d) - (1 + 50d)| + |(1 + 3d) - (1 + 50d)| + \dots + |(1 + 99d) - (1 + 50d)|)$$

$$= \frac{1}{50} (49d + 47d + \dots + d + d + 3d + \dots + 49d)$$

$$= \frac{d}{25} (1 + 3 + \dots + 49)$$

$$= \frac{d}{25} \cdot \frac{25}{2} (50)$$

$$= 25d = 250 \quad \text{(Given)}$$

$$\Rightarrow d = 10$$

76. Answer (1)

$$2f(x) = f\left(\frac{1}{x}\right) \cdot x + 1 \quad \dots(i)$$

Replacing  $x$  by  $\frac{1}{x}$ ,

$$2f\left(\frac{1}{x}\right) = f(x) \cdot \frac{1}{x} + 1 \quad \dots(ii)$$

From (i) and (ii),

$$3f(x) = 2x + \frac{1}{x} + 1$$

$$y = xf(x) = \frac{1}{3}(2x^2 + x + x)$$

$$\frac{dy}{dx} = \frac{1}{3}(4x + 1)$$

$$\left. \frac{dy}{dx} \right|_{\text{at } x=1} = \frac{5}{3}$$

77. Answer (2)

$p$	$q$	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \vee p$	$(p \wedge q)$	$p \rightarrow (p \vee q)$	$p \rightarrow (p \wedge q)$
T	T	T	T	T	T	T	T
T	F	F	T	T	F	T	F
F	T	F	T	T	F	T	T
F	F	T	T	F	F	T	T

$$\Rightarrow p \rightarrow (q \rightarrow p) \text{ is equivalent to } p \rightarrow (p \vee q)$$

78. Answer (2)

$$\log_{0.09}(x-2) \geq \log_{0.3}(x-2)$$

$$\Rightarrow \frac{\ln(x-2)}{\ln(0.3)^2} \geq \frac{\ln(x-2)}{\ln(0.3)}$$

$$\Rightarrow \frac{\ln(x-2)}{2} \leq \ln(x-2)$$

$$\Rightarrow (x-2) \leq (x-2)^2$$

$$\Rightarrow (x-2)(x-2-1) \geq 0$$

$$\Rightarrow (x-2)(x-3) \geq 0$$

But,  $x > 2$  and  $x > 3$

79. Answer (3)

Mean of 1<sup>st</sup> data =  $\frac{x_1 + x_2 + \dots + x_6}{6} = 9$

$$\Rightarrow \sum_{i=1}^6 x_i = 54$$

Mean of 2<sup>nd</sup> data =  $\frac{x_7 + x_8 + x_9}{3} = 9$

$$\Rightarrow \sum_{i=7}^9 x_i = 27$$

Mean of the combined data

$$= \bar{x} = \frac{x_1 + x_2 + \dots + x_9}{9} = 9$$

Variance of 1<sup>st</sup> data =  $\frac{\sum_{i=1}^6 (x_i - 9)^2}{6} = 27$

Variance of 2<sup>nd</sup> data =  $\frac{\sum_{i=7}^9 (x_i - 9)^2}{3} = 27$

$\therefore$  Variance of the combined data

$$= \frac{\sum_{i=1}^9 (x_i - \bar{x})^2}{9}$$

$$= \frac{\sum_{i=1}^6 (x_i - 9)^2 + \sum_{i=7}^9 (x_i - 9)^2}{9}$$

$$= \frac{27 \cdot 6 + 27 \cdot 3}{9}$$

$$= 27$$

80. Answer (2)

p	q	p → q	~(p → q)	p ∨ ~q	p ∧ ~q	~p → ~q	p ∧ q
T	T	T	F	T	F	T	T
T	F	F	T	T	T	T	F
F	T	T	F	F	F	F	F
F	F	T	F	T	F	T	F

81. Answer (1)

$$\lim_{x \rightarrow 0} \frac{(2^x - 1)^2}{\sin(x) \ln \left(1 + \frac{x}{2}\right)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{2^x - 1}{x} \cdot \frac{2^x - 1}{x}}{\frac{\sin x}{x} \cdot x \ln \left(1 + \frac{x}{2}\right)} \cdot \frac{2}{x}$$

$$= \frac{2}{x} (\ln 2)^2$$

$$= (\ln 2)^2 \quad (\text{Given})$$

$$\Rightarrow \lambda = 2$$

82. Answer (3)

$$\lim_x \frac{\sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x}}{\sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x}}$$

$$= \lim_x \frac{\sqrt{x} \sqrt{x}}{\sqrt{x} \sqrt{x} \sqrt{x} \sqrt{x}}$$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{x} \left( \sqrt{1 + \sqrt{\frac{1}{x}}} \right)}{\sqrt{x} \left( \sqrt{1 + \sqrt{\frac{1}{x} + \sqrt{\frac{1}{x^3} + 1}} \right)}$$

$$= \frac{1}{2}$$

83. Answer (3)

Let the joining of (2, 3, 7) and (3, 4, -1) be divided by xy plane in the ratio  $\lambda : 1$ .

z-coordinate = 0

$$\Rightarrow \frac{7}{1} = 0$$

$$\Rightarrow \lambda = 7$$

84. Answer (4)

$$\ln \frac{x^3 - y^3}{x^3 + y^3}$$

$$\Rightarrow \frac{x^3 - y^3}{x^3 + y^3} = e$$

Differentiating,

$$(x^3 + y^3)(3x^2 - 3y^2y') - (x^3 - y^3)(3x^2 + 3y^2y') = 0$$

$$\Rightarrow (x^3 + y^3)x^2 - (x^3 + y^3)y^2 - (x^3 - y^3)x^2 - (x^3 - y^3)y^2 = 0$$

$$\Rightarrow y\{x^3y^2 + y^5 + x^3y^2 - y^5\} = x^5 + x^2y^3 - x^5 + x^2y^3$$

$$\Rightarrow y \frac{2x^2y^3}{2x^3y^2} \frac{y}{x}$$

85. Answer (1)

$$\lim_{x \rightarrow \infty} \frac{\ln [2x]}{x}$$

$$2x - 1 < [2x] \leq 2x$$

$$\text{Now, } \lim_{x \rightarrow \infty} \frac{\log(2x-1)}{x} = \lim_{x \rightarrow \infty} \frac{1}{2x-1} = 0$$

$$\lim_{x \rightarrow \infty} \frac{\log 2x}{x} = \lim_{x \rightarrow \infty} \frac{1}{2x} = 0$$

$$\therefore \lim_{x \rightarrow \infty} \frac{[2x]}{x} = 0$$

86. Answer (1)

p	q	r	p ∧ q	q ∧ r	(p ∧ q) ∧ (q ∧ r)
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	F
T	F	F	F	F	F
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

∴ Statement-1 is also true and statement-2 is correct explanation of statement-1.

87. Answer (1)

Let  $p$  : I complete the syllabus today.  
 $q$  : I shall go to give test tomorrow.  
 $r$  : I shall get good marks in the test tomorrow.  
 The assumptions are  $p \Rightarrow q$ ,  $q \Rightarrow r$  and are conclusion is  $p \Rightarrow r$ .  
 ∴ Both statements are correct and statement-2 is the correct explanation of statement-1.

88. Answer (4)

Statement-2 is true.

**Statement-1 :**

$$\text{LHL} = \lim_{h \rightarrow 0} \frac{e^{2/h} - e^{2/h}}{e^{2/h} - e^{2/h}}$$

$$= \lim_{h \rightarrow 0} \frac{e^{4/h} - 1}{e^{4/h} - 1}$$

$$= -1$$

$$\text{RHL} = \lim_{h \rightarrow 0} \frac{e^{2/h} - e^{2/h}}{e^{2/h} - e^{2/h}}$$

$$= \lim_{h \rightarrow 0} \frac{1 - e^{4/h}}{1 - e^{4/h}}$$

$$= 1$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{e^{2/x} - e^{2/x}}{e^{2/x} - e^{2/x}} \text{ does not exist}$$

89. Answer (3)

Statement-2 is false as correct statement is

$$rx - 1 < [rx] \leq rx, r = 1, 2, 3, \dots, n$$

$$\Rightarrow (1 \ 2 \ \dots \ n)x \leq n \binom{n}{r-1} [rx] \leq (1 \ 2 \ \dots \ n)x$$

$$\Rightarrow \frac{n(n-1)}{2}x \leq n \binom{n}{r-1} [rx] \leq \frac{n(n-1)}{2}x$$

$$\Rightarrow 1 \frac{1}{n} \frac{x}{2} \leq \frac{1}{n} \frac{[rx]}{n^2} \leq 1 \frac{1}{n} \frac{x}{2}$$

Applying Sandwich theorem,

$$\lim_{n \rightarrow \infty} \frac{[rx]}{n^2} = \frac{x}{2}$$

90. Answer (3)

Statement-2 is false  $x^2 \frac{1}{x^2} \geq 2, x \in \mathbb{R} \setminus \{0\}$ .

**Statement-1 :**

$$\text{LHS} = 3 \sin x + 4 \cos x \in [-5, 5]$$

$$\text{RHS} = y^2 - 10y + 30$$

$$= (y - 5)^2 + 5 \geq 5$$

∴ Solution(s) exist(s) only when LHS = RHS = 5

$$\Rightarrow 3 \sin x + 4 \cos x = 5 \quad \text{and} \quad (y - 5)^2 + 5 = 5$$

$$\sin(x + \alpha) = 1, \text{ where } \tan \frac{4}{3} \Rightarrow y = 5$$

$$\therefore x \in [0, \pi]$$

∴ It is satisfied for only one value of  $x$ .

