

TEST-12**ANSWERS****PHYSICS**

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30. (4)

CHEMISTRY

31. (4)
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33. (4)
34. (4)
35. (1)
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MATHEMATICS

61. (1)
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64. (4)
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88. (4)
89. (1)
90. (1)

Answers & Hints

PART - A : (PHYSICS)

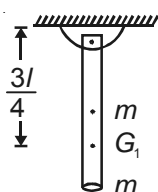
1. Answer (2)

$$d = \frac{3l}{4}$$

$$\text{and } I = \frac{1}{3}ml^2 + ml^2 = \frac{4}{3}ml^2$$

$$\Rightarrow T = 2\pi\sqrt{\frac{4ml^2/3}{2mgd}} \Rightarrow T = 2\pi\sqrt{\frac{4ml^2/3}{2mg \times 3l/4}}$$

$$T = \frac{8\pi}{3}\sqrt{\frac{l}{2g}}$$



2. Answer (1)

$$\frac{1}{2}kx_0^2 = \frac{1}{2}\mu v_r^2$$



$$\mu = \frac{m \times 3m}{4m} \Rightarrow \mu = \frac{3m}{4}$$

$$\frac{1}{2}kx_0^2 = \frac{1}{2} \times \left(\frac{3m}{4}\right) v_r^2 \Rightarrow v_r = \sqrt{\frac{4k}{3m}} x_0$$

3. Answer (1)

$$U = k \log r$$

$$\Rightarrow F = -\frac{dU}{dr} = -\frac{k}{r}$$

$$\Rightarrow \frac{k}{r} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{k}{m}}$$

By Bohr's quantization rule

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow m\sqrt{\frac{k}{m}}r = \frac{nh}{2\pi} \Rightarrow r \propto n$$

4. Answer (1)

$$u = u_1 + u_2$$

$$\Rightarrow (n_1 + n_2)c_v T = n_1 C_{v1} T_1 + n_2 C_{v2} T_2$$

$$\Rightarrow (n_1 C_{v1} + n_2 C_{v2})T = n_1 C_{v1} T_1 + n_2 C_{v2} T_2$$

$$\Rightarrow T = \frac{n_1 C_{v1} T_1 + n_2 C_{v2} T_2}{n_1 C_{v1} + n_2 C_{v2}}$$

$$\Rightarrow T = \frac{\frac{3}{2}n_1 T_1 + \frac{5}{2}n_2 T_2}{\frac{3}{2}n_1 + \frac{5}{2}n_2} \Rightarrow T = \frac{3n_1 T_1 + 5n_2 T_2}{3n_1 + 5n_2}$$

5. Answer (2)

$$y = A \sin kx \sin \omega t$$

For fundamental mode,

$$\frac{\lambda}{2} = L \Rightarrow \lambda = 2L$$

$$\Rightarrow k = \frac{2\pi}{\lambda} \Rightarrow k = \frac{\pi}{L}$$

$$\text{Also, } \omega = 2\pi f \Rightarrow \omega = \frac{\pi v}{L}$$

$$y = A \sin\left(\frac{\pi}{L}x\right) \sin\left[\frac{\pi}{L}\sqrt{\frac{T}{\mu}}t\right]$$

6. Answer (2)

$$pv^2 = \text{constant} \quad \text{---(1)}$$

$$pv = nRT \quad \text{---(2)}$$

From (1) & (2),

$$\left(\frac{nRT}{v}\right)v^2 = \text{const.} \Rightarrow TV = \text{const.}$$

$$\Rightarrow TdV + VdT = 0 \Rightarrow TdV = -VdT$$

$$\Rightarrow \gamma = \frac{1}{V}\left(\frac{dV}{dT}\right) \Rightarrow \gamma = -\frac{1}{T}$$

7. Answer (1)

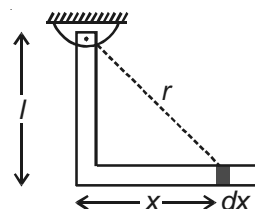
$$r^2 = \rho + x^2$$

$$dl_1 = dm\rho$$

$$dl_1 = \left(\frac{m}{l}\right)(\rho + x^2)dx$$

$$l_1 = \frac{m}{l} \int_0^l (\rho^2 + x^2) dx$$

$$\Rightarrow l_1 = \frac{4}{3}m\rho$$



$$l = l_1 + l_2$$

$$= \frac{4}{3}m\rho + \frac{1}{3}m\rho \Rightarrow \frac{5}{3}m\rho$$

$$\text{By } \tau = l\alpha, mg \frac{l}{2} = \frac{5}{3}m\rho\alpha \Rightarrow \alpha = \frac{3g}{10l}$$

8. Answer (3)

$$u \propto \sqrt{P} \Rightarrow u = c\sqrt{P}$$

$$\Rightarrow nC_v T = C\sqrt{P} \Rightarrow \sqrt{P} \propto T \Rightarrow \frac{P}{T^2} = \text{const. and}$$

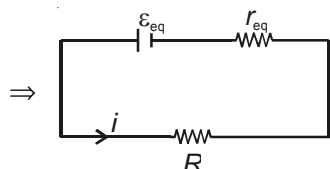
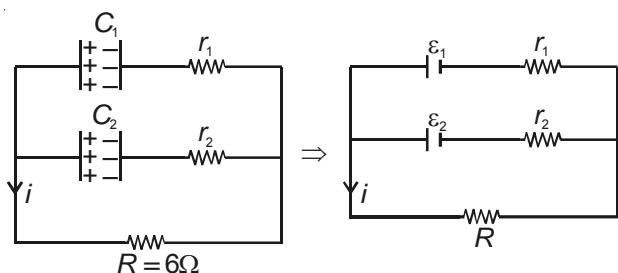
$$PV = nRT \Rightarrow T = \frac{PV}{nR}$$

$$\frac{Pn^2R^2}{\rho^2V^2} = \text{const.} \Rightarrow PV^2 = \text{const.}$$

$$C = C_v + \frac{R}{1-2}$$

$$\Rightarrow C = \frac{3R}{2} - R \Rightarrow C = \frac{R}{2}$$

9. Answer (3)



$$\epsilon_1 = \frac{Q_1}{C_1} \Rightarrow \epsilon_1 = 5V, r_1 = 1\Omega$$

$$\epsilon_2 = \frac{Q_2}{C_2} \Rightarrow \epsilon_2 = 2V, r_2 = 2\Omega$$

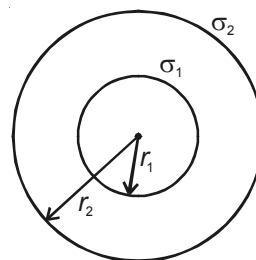
$$\epsilon_{eq} = \frac{\epsilon_1 r_2 + \epsilon_2 r_1}{r_1 + r_2} \Rightarrow \epsilon_{eq} = \frac{5 \times 2 + 2 \times 1}{2 + 1}$$

$$\Rightarrow \epsilon_{eq} = 4V$$

$$\text{Also, } r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \Rightarrow r_{eq} = \frac{2}{3}$$

$$\Rightarrow i = \frac{\epsilon_{eq}}{R + r_{eq}} \Rightarrow i = \frac{4}{6 + \frac{2}{3}} = 0.6 \text{ A}$$

10. Answer (1)



$$q_1 = (4\pi r_1^2)\sigma_1 \quad q_2 = (4\pi r_2^2)\sigma_2$$

$$V_1 - V_2 = \frac{q_1}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$= \frac{\sigma_1 r_1^2}{\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$V_1 - V_2 = \frac{\sigma_1 r_1 (r_2 - r_1)}{\epsilon_0 r_2}$$

11. Answer (3)

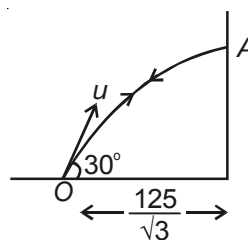
$$E = K + U$$

$$-\frac{GMm}{r_1 + r_2} = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$\Rightarrow \frac{1}{2}mv^2 = GMm \left[\frac{1}{r} - \frac{1}{r_1 + r_2} \right]$$

$$\Rightarrow v = \sqrt{2GM \left[\frac{1}{r} - \frac{1}{r_1 + r_2} \right]}$$

12. Answer (1)



For, O to A

$$u_x \times t_1 = \frac{125}{\sqrt{3}} \quad \text{---(1)}$$

For, A to O,

$$u'_x \times t_2 = \frac{125}{3} \quad \text{---(2)} \quad (u'_x = eu_x)$$

$$t_1 + t_2 = T \Rightarrow t_1 + t_2 = \frac{2u_y}{g}$$

$$\Rightarrow t_1 + t_2 = 5 \quad \text{---(3)}$$

On solving (1), (2) and (3)

$$e = \frac{1}{2}$$

13. Answer (1)

$$\Delta P = -B \left(\frac{\partial y}{\partial x} \right) \Rightarrow \Delta P = -B(\text{Slope of wave})$$

14. Answer (2)

$$\vec{p} = p\hat{i}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{x^2} \right) \hat{i}$$

As $u = -\vec{p} \cdot \vec{E}$

$$\Rightarrow u = -\frac{1}{4\pi\epsilon_0} \left(\frac{Qp}{x^2} \right)$$

$$\Rightarrow F = -\frac{\partial u}{\partial x}$$

$$\Rightarrow F = -\left[-\left(\frac{Qp}{4\pi\epsilon_0} \right) \left(\frac{-2}{x^3} \right) \right]$$

$$\Rightarrow \vec{F} = \frac{1}{4\pi\epsilon_0} \left(\frac{2pQ}{a^3} \right) (-\hat{i})$$

15. Answer (1)

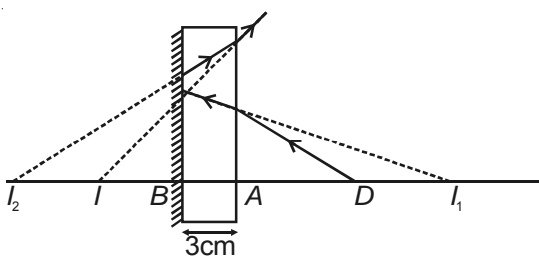
$$Al_1 = 1.5 \times AO$$

$$\Rightarrow Al_1 = 1.5 \times 10$$

$$Al_1 = 15 \text{ cm}$$

$$Bl_1 = Al_1 + AB$$

$$\Rightarrow Bl_1 = 18 \text{ cm}$$



$$Bl_2 = Bl_1 = 18 \text{ cm}$$

$$Al_2 = Bl_2 + AB$$

$$\Rightarrow Al_2 = 21 \text{ cm}$$

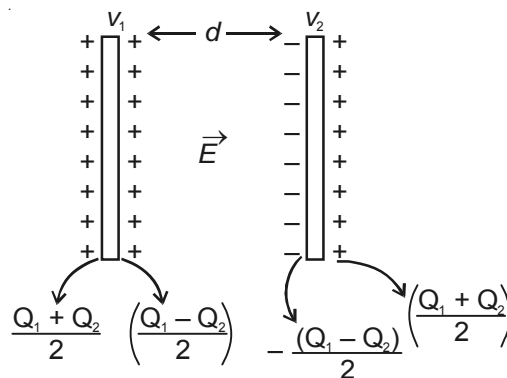
$$Al = \frac{1}{1.5} \times 21 \text{ cm} \Rightarrow 14 \text{ cm}$$

$$Ol = Al + AO$$

$$\Rightarrow Ol = 14 + 10$$

$$Ol = 24 \text{ cm}$$

16. Answer (4)



$$E = \frac{(Q_1 - Q_2) / 2A}{\epsilon_0}$$

$$V_1 - V_2 = Ed$$

$$\Rightarrow V_1 - V_2 = \frac{(Q_1 - Q_2) d}{2\epsilon_0 A}$$

17. Answer (2)

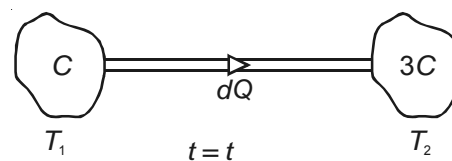
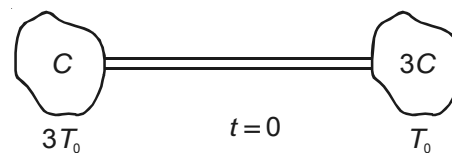
18. Answer (3)

19. Answer (4)

20. Answer (1)

21. Answer (3)

22. Answer (1)



$$dQ = -CdT_1 \quad \text{---(1)}$$

$$dQ = 3CdT_2 \Rightarrow \frac{dQ}{3} = CdT_2 \quad \text{---(2)}$$

(1) + (2)

$$\frac{4}{3} dQ = C(dT_2 - dT_1) \Rightarrow \frac{4}{3} dQ = -Cd(T_1 - T_2)$$

$$\frac{4}{3} dQ = -Cd\theta \quad (\theta \rightarrow \text{temp diff.}) \quad \text{---(3)}$$

$$\frac{dQ}{dt} = \frac{kA}{l} (T_1 - T_2)$$

$$-\frac{3C}{4} \left(\frac{d\theta}{dt} \right) = \left(\frac{kA}{l} \right) \theta$$

$$\Rightarrow \int_{\theta_0}^{\theta} \frac{d\theta}{\theta} = \int_0^t -\frac{4kA}{3Cl} dt$$

$$\Rightarrow \theta = \theta_0 e^{-\frac{4kA}{3Cl} t} \quad \theta_0 = 3T_0 - T_0$$

$$\Rightarrow \theta = 2T_0 e^{-\left(\frac{4kA}{3Cl}\right)t}$$

23. Answer (2)

From (1) and (2)

$$-CdT_1 = 3CdT_2 \Rightarrow -dT_1 = 3dT_2$$

$$\Rightarrow T_{1i} - T_{1f} = 3(T_{2f} - T_{2i}) \Rightarrow 3T_0 - T_1 = 3(T_2 - T_0)$$

$$\Rightarrow T_1 + 3T_2 = 6T_0 \quad \text{---(4)}$$

$$\text{and } T_1 - T_2 = 2T_0 e^{-\alpha t} \quad \text{---(5)}$$

$$\alpha = \left(\frac{4kA}{3l} \right)$$

solving (4) & (5)

$$T_1 = \frac{3}{2} T_0 (1 + e^{-\alpha t})$$

24. Answer (3)

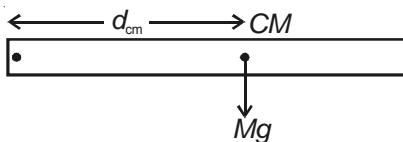
$$T_1 = \frac{3}{2} T_0 (1 + e^{-\alpha t})$$

at steady state, $t \rightarrow \infty$

$$\Rightarrow T_1 = \frac{3T_0}{2}$$

25. Answer (4)

$$d_{cm} = \frac{\int x dm}{\int dm}$$



$$d_{cm} = \frac{\int_0^l x \left(\frac{\lambda_0}{l} \right) x dx}{\int_0^l \left(\frac{\lambda_0}{l} \right) x dx}$$

$$d_{cm} = \frac{\int_0^l x^2 dx}{\int_0^l x dx} \Rightarrow d_{cm} = \frac{l^3/3}{l^2/2} \Rightarrow d_{cm} = \frac{2l}{3}$$

$$M = \int dm \Rightarrow \int_0^l \left(\frac{\lambda_0}{l} \right) x dx \Rightarrow M = \frac{\lambda_0 l}{2}$$

$$dl = dm x^2 \Rightarrow l = \int_0^l \left(\frac{\lambda_0}{l} \right) x^3 dx \Rightarrow l = \frac{\lambda_0 l^3}{4}$$

$$\tau = l\alpha \Rightarrow Mg \times \frac{2l}{3} = l\alpha$$

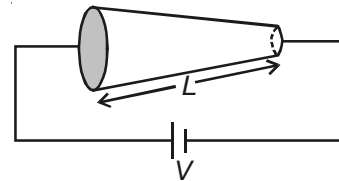
$$\Rightarrow \frac{\lambda_0 l}{2} g \times \frac{2l}{3} = \frac{\lambda_0 l^3}{4} \alpha$$

$$\Rightarrow a = \frac{4g}{3l}$$

$$a_{cm} = d_{cm} \times \alpha$$

$$\Rightarrow a_{cm} = \frac{8}{9} g$$

26. Answer (1)



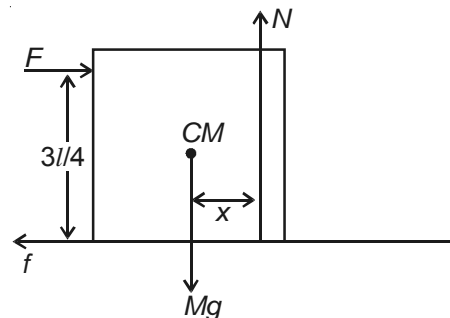
$$R = \frac{\rho L}{\sqrt{A_1 A_2}} \quad A_1 = A, A_2 = \frac{A}{4}$$

$$R = \frac{\rho L}{A/2} \Rightarrow R = \frac{2\rho L}{A}$$

$$i = \frac{V}{R} \Rightarrow i = \frac{VA}{2\rho L} \quad \text{---(i)}$$

$$J = \frac{i}{A} = \frac{8V}{9\rho L}$$

27. Answer (3)



For translational equilibrium

$$f = F$$

and $f \leq f_s$

$$F \leq f_s \Rightarrow 2Mgt \leq \mu Mg$$

$$\Rightarrow t \leq \frac{\mu}{2} \Rightarrow t \leq 0.4s$$

For rotational equilibrium

$$F \times \frac{l}{4} + f \times \frac{l}{2} = Nx$$

$$\Rightarrow F \times \frac{3l}{4} = Mg x \Rightarrow x = \frac{3Fl}{4Mg} \Rightarrow x = \left(\frac{3l}{2}\right)t$$

$$x \leq \frac{l}{2} \Rightarrow \left(\frac{3l}{2}\right)t \leq \frac{l}{2} \Rightarrow t \leq \frac{1}{3} \Rightarrow t \leq 0.33s$$

So block will topple before slipping

28. Answer (2)

For lens

$$\frac{1}{f_L} = (\mu - 1) \left[\frac{1}{R} - \frac{1}{\infty} \right]$$

$$\Rightarrow P_L = \frac{1}{f_2} \Rightarrow P_L = \frac{(\mu - 1)}{R}$$

For mirror

$$f_m = \infty \Rightarrow P_m = 0$$

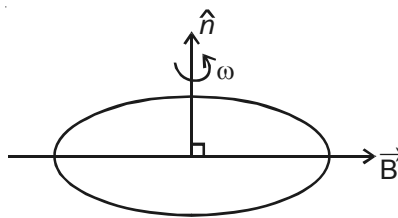
Power of the system

$$P = 2P_L + P_m$$

$$\Rightarrow P = \frac{2(\mu - 1)}{R} \Rightarrow P = \frac{2 \times 0.5}{0.2} \Rightarrow P = 5 \text{ D}$$



29. Answer (2)



$$\frac{M}{L} = \frac{q}{2m} \Rightarrow M = \left(\frac{q}{2m}\right)l\omega$$

$$M = \frac{q}{2m} \times \frac{1}{2} m R^2 \times \omega \Rightarrow \vec{M} = \left(\frac{q\omega R^2}{4}\right)\hat{n}$$

$$mg \times R \geq |\vec{M} \times \vec{B}| \Rightarrow mgR \geq \frac{q\omega R^2 B}{4}$$

$$\Rightarrow B \leq \frac{4mg}{q\omega R}$$

$$B_{\max} = \frac{4mg}{q\omega R}$$

30. Answer (4)

$$Q_0 = CV_0$$

$$i = \omega \sqrt{Q_0^2 - q^2}$$

$$i_{\max} = Q_0 \omega, i = \frac{i_{\max}}{2} \Rightarrow i = \frac{Q_0 \omega}{2}$$

$$\frac{Q_0 \omega}{2} = \omega \sqrt{Q_0^2 - q^2}$$

$$\Rightarrow q = \frac{\sqrt{3}}{2} Q_0$$

$$\Rightarrow V = \frac{\sqrt{3}}{2} V_0$$

$$L \left| \frac{di}{dt} \right| = V$$

$$\Rightarrow \left| \frac{di}{dt} \right| = \frac{\sqrt{3} V_0}{2L}$$

PART - B : (CHEMISTRY)

31. Answer (4)

$$\text{Equivalent wt. of metal} = \frac{0.40}{1.2 - 0.4} \times 8 = 4$$

32. Answer (1)

$$-3, +3, -3$$

33. Answer (4)

$$\text{Rate of diffusion} \propto \frac{1}{\text{molar.mass}}$$

$$\text{NH}_3 > \text{CO}_2 > \text{SO}_2$$

34. Answer (4)

$$\text{pH} = 13.3$$

$$[\text{OH}^-] = 0.1 \times 2 = 0.2 \text{ M}$$

$$\text{pOH} = -\log[2 \times 10^{-1}]$$

$$= -(\log 2 - 1 \log 10)$$

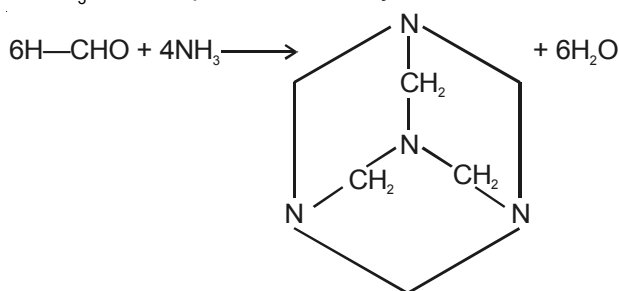
$$= -(0.3010 - 1)$$

$$= 0.699$$

$$\text{pH} = 14 - 0.699 = 13.3$$

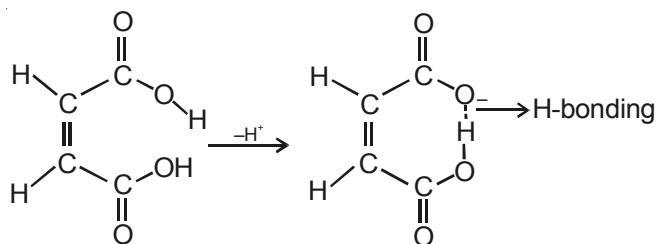
35. Answer (1)
n-pentane
B.P. \propto length of the chain.
36. Answer (1)
 $C_2H_5 - NO_2$
 $AgNO_2 + C_2H_5Cl \longrightarrow AgCl + C_2H_5NO_2$
37. Answer (2)
Reactivity of alcohol \propto ease of release of OH^- group
 \propto stability of carbocation formed.
 $III > IV > II > I$

38. Answer (1)
Compound obtained by reaction of formaldehyde with NH_3 is Urotropin or hexamethylene tetramine.



39. Answer (4)
40. Answer (4)
 $(CH_3)_3N$ because of no active hydrogen.
41. Answer (2)
ATP
42. Answer (4)
Both (1) & (3)
43. Answer (4)
CoS
44. Answer (3)
Zn
45. Answer (1)
 SO_3
46. Answer (1)
 $CaCO_3$
47. Answer (1)
Statement-1 is True, Statement-2 is True;
Statement-2 is a correct explanation for Statement-1.
48. Answer (2)
Statement-1 is True, Statement-2 is True;
but Statement-2 is **NOT** a correct explanation for Statement-1.
49. Answer (1)
Statement-1 is True, Statement-2 is True;
Statement-2 is a correct explanation for Statement-1.
50. Answer (1)
Statement-1 is True, Statement-2 is True;
Statement-2 is a correct explanation for Statement-1.

51. Answer (1)
Statement-1 is True, Statement-2 is True;
Statement-2 is a correct explanation for Statement-1.
52. Answer (3)
HCl
53. Answer (1)
Glycerol > glycol > ethyl alcohol
54. Answer (1)
Maleic acid > Fumaric acid



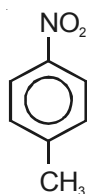
55. Answer (4)
Both (1) & (3)
Hinderson equation.

$$pH = pK_a + \log \frac{[Salt]}{[Acid]}$$

$$\text{or } pH = -\log K_a + \log \frac{[Salt]}{[Acid]}$$

$$\text{or } = -\log K_a - \log \frac{[Acid]}{[Salt]}$$

56. Answer (3)



57. Answer (4)
58. Answer (3)
 $CH_3-\overset{\oplus}{C}H-OH$ due to +R effect of $-OH$ group and +I effect of CH_3 .
59. Answer (1)
 due to effective sidewise overlapping of filled 2p of F and π 2p orbital of benzene ring.
60. Answer (2)
Diekmann condensation.

PART - C : (MATHEMATICS)

61. Answer (1)

$$x = 3t^2 + 2 \text{ and } y = t + 1$$

$$\Rightarrow x = 3(y-1)^2 + 2$$

$$\Rightarrow x - 2 = 3(y-1)^2$$

\therefore Parabola with vertex (2, 1)

62. Answer (1)

Differentiating, keeping y as constant

$$3f' \left(\frac{3x+5y}{4} \right) = \frac{3f'(x)}{4}$$

$$\text{Put } x = 0, y = \frac{4}{5}x$$

$$\Rightarrow 3f'(x) = \frac{3}{4}f'(0)$$

$$\Rightarrow f'(x) = \frac{1}{2}$$

$$\Rightarrow f(x) = \frac{x}{2} + K$$

63. Answer (2)

$$\text{Im}(x^2 - y^2 + 2ixy) = 4$$

$$\Rightarrow xy = 2$$

Rectangular hyperbola

64. Answer (4)

$$\text{If } x \leq 3, \quad x^2 + 1 \leq 6 - 2x$$

$$\Rightarrow (x+1)^2 \leq 6$$

$$\Rightarrow -\sqrt{6}-1 \leq x \leq \sqrt{6}-1$$

$$\text{If } x > 3 \Rightarrow x^2 + 1 \leq 2x - 6$$

$$\Rightarrow (x-1)^2 \leq -6 \text{ not possible}$$

65. Answer (4)

$${}^{n+1}C_5 > {}^{n+1}C_4$$

$$\Rightarrow \frac{1}{5!(n-4)!} > \frac{1}{4!(n-3)!}$$

$$\Rightarrow \frac{1}{5} > \frac{1}{n-3}$$

$$\Rightarrow 5 < n-3$$

$$\Rightarrow n > 8$$

$$\therefore n = 9, 10, 11$$

66. Answer (1)

$$\sum_{r=1}^{\infty} \tan^{-1} \left\{ \frac{r^2 - (r-1)^2}{1 + r^2(r-1)^2} \right\}$$

$$S_n = \sum_{r=1}^n \tan^{-1} r^2 - \tan^{-1} (r-1)^2$$

$$S_n = (\tan^{-1} 1 - \tan^{-1} 0) + (\tan^{-1} 2^2 - \tan^{-1} 1^2) + \dots + \{\tan^{-1} n^2 - \tan^{-1} (n-1)^2\}$$

$$\therefore S_{\infty} = \frac{\pi}{2}$$

67. Answer (3)

$$a = 8; \quad b = 10; \quad c = 12$$

$$\cos A = \frac{100 + 144 - 64}{240} = \frac{3}{4}$$

$$\therefore \sin A = \frac{\sqrt{7}}{4}$$

$$2R = \frac{a}{\sin A} = \frac{8}{\frac{\sqrt{7}}{4}}$$

$$= \frac{32\sqrt{7}}{7}$$

68. Answer (2)

$$f(x) = 4^{(x-2)^2} - 4$$

$$\therefore \text{Range} = \left[\frac{1}{256}, \infty \right)$$

69. Answer (1)

$$[x]^2 \geq x$$

$$\Rightarrow x \in \mathbb{R} - \{(0, 1) \cup (1, 2)\}$$

70. Answer (3)

$$17^{100} = (1+16)^{100}$$

$$= 1 + {}^{100}C_1 \cdot 16 + {}^{100}C_2 \cdot 16^2 + \dots + {}^{100}C_{100} \cdot 16^{100}$$

$$\therefore \text{Remainder} = 1$$

71. Answer (2)

$$\frac{\sqrt{2}}{\cos\theta + \sin\theta} - \frac{\sqrt{2}}{\cos\theta - \sin\theta} = \sqrt{2}$$

$$\Rightarrow \frac{-2\sin\theta}{1 - 2\sin^2\theta} = 1$$

$$\Rightarrow -2\sin\theta = 1 - 2\sin^2\theta$$

$$\Rightarrow 2\sin^2\theta - 2\sin\theta - 1 = 0$$

$$\sin\theta = \frac{2 \pm \sqrt{4+8}}{2 \times 2}$$

$$\sin\theta = \frac{1 \pm \sqrt{3}}{2}$$

$$\text{Acceptable value is } \sin\theta = \frac{1 - \sqrt{3}}{2}$$

∴ No. of roots = 4

72. Answer (3)

Mode

73. Answer (2)

Normal at $(am^2, -2am)$ is $y = mx - 2am - am^3$

$$\therefore am^2 = 20 \text{ and } -2am = 20$$

$$\Rightarrow m = -2 \text{ and } a = 5$$

$$\therefore \text{Normal is } y = -2x + 20 + 40$$

$$2x + y = 60$$

74. Answer (1)

$$\max \{3 \sin x - 4 \cos x\} = 5$$

∴ No solution

75. Answer (3)

$x = y$ satisfies the equation

76. Answer (3)

Let x_1, x_2, x_3, x_4 are roots

Then

$$\sum x_1 x_2 = -1$$

77. Answer (1)

$$\frac{x^2}{2} + \frac{y^2}{3} = 1, (x-1)^2 + (y+2)^2 = 1$$

∴ Length of minor axis = $2\sqrt{2}$

diameter = 2

∴ Area of ellipse > area of circle

78. Answer (4)

Let $|\vec{a}| = |\vec{b}| = r$

$$\vec{a} \cdot \vec{b} = \frac{r^2}{2}$$

$$\vec{a} \cdot (\vec{a} + \vec{b}) = r^2 + \frac{r^2}{2} \neq 0$$

∴ \vec{a} is not perpendicular to $\vec{a} + \vec{b}$.

Statement 1 is false and statement 2 is true.

79. Answer (4)

$$\text{If } x > 0, I = \int x^3 dx = \frac{x^4}{4} + C$$

$$\text{If } x < 0, I = -\int x^3 dx = -\frac{x^4}{4} + C$$

∴ Statement 1 is false

80. Answer (1)

$$\therefore \theta, \alpha \in \left(0, \frac{\pi}{2}\right) \Rightarrow \sec\theta > 1 \text{ and } \operatorname{cosec}\alpha > 1$$

∴ Statement 2 is correct explanation of Statement-1.

81. Answer (3)

$$\frac{\tan x}{x} > 1 \Rightarrow \lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right] = 1$$

But $\tan x > x \Rightarrow x > \tan^{-1} x$

$$\therefore \lim_{x \rightarrow 0} \left[\frac{\tan^{-1} x}{x} \right] = 0$$

82. Answer (4)

$$\left. \begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} \frac{2 - 3e^{1/x}}{5 + 2e^{1/x}} = \frac{2}{5} \\ \text{RHL} &= \lim_{x \rightarrow 0^+} \frac{2e^{-1/x} - 3}{5e^{-1/x} + 2} = -\frac{3}{2} \end{aligned} \right\} \text{LHL} \neq \text{RHL}$$

83. Answer (2)

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} \frac{x(2 - 3e^{1/x})}{5 + 2e^{1/x}} \\ &= 0 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} \frac{x(2e^{-1/x} - 3)}{5e^{-1/x} + 2} \\ &= 0 \end{aligned}$$

$$f(0) = 0$$

$\therefore f(x)$ is continuous at $x = 0$

$$\begin{aligned} \text{LHD} &= \lim_{x \rightarrow 0^-} \frac{x(2 - 3e^{1/x})}{5 + 2e^{1/x}} \times \frac{1}{x} \\ &= \frac{2}{5} \end{aligned}$$

$$\begin{aligned} \text{RHD} &= \lim_{x \rightarrow 0^+} \frac{x(2e^{-1/x} - 3)}{5e^{-1/x} + 2} \times \frac{1}{x} \\ &= -\frac{3}{2} \end{aligned}$$

$\therefore f'(0)$ does not exist

84. Answer (4)

$$\text{Let } g(x) = e^{-1/x} f(x)$$

$$\Rightarrow g(x) = \frac{2e^{-1/x} - 3}{5 + 2e^{1/x}}$$

$$\text{LHL} = \lim_{x \rightarrow 0^-} g(x) \rightarrow \infty$$

$$\text{RHL} = \lim_{x \rightarrow 0^+} g(x) \rightarrow 0$$

$\therefore \lim_{x \rightarrow 0} g(x)$ does not exist.

85. Answer (1)

$$a_1 = (3.5.7.9.11 - 1.3.5.7.9) \frac{1}{10}$$

$$a_2 = (5.7.9.11.13 - 3.5.7.9.11) \frac{1}{10}$$

$$a_{10} = (21.23.25.27.29 - 19.21.23.25.27) \frac{1}{10}$$

$$\Rightarrow S_{10} = \frac{1}{10} \{9454725 - 945\}$$

$$\Rightarrow S_{10} = 945378$$

86. Answer (2)

$$f(x) + f(x+2) + f(x+4) = 10$$

$$f(x+2) + f(x+4) + f(x+6) = 10$$

$$f(x) = f(x+6)$$

$$\begin{aligned} \therefore \int_2^{62} f(x) dx &= 10 \int_0^6 f(x) dx \\ &= 10 I \end{aligned}$$

87. Answer (2)

$$f(x) = f^{-1}(x) \Rightarrow (f \circ f)(x) = x$$

$$\Rightarrow (x^2 - x + 1)^2 - x^2 + x - 1 + 1 = x$$

$$\Rightarrow x^4 + 1 - 2x^3 - 2x + 2x^2 = 0$$

$$\Rightarrow x^4 - 2x^3 + 2x^2 - 2x + 1 = 0$$

$$\Rightarrow (x-1)^2 (x^2 + 1) = 0$$

$$\Rightarrow x = 1, 1$$

88. Answer (4)

$$R(x) = \int \frac{e^x (x^2 - x + 1)}{(x^2 + 1)^{3/2}} dx$$

$$R(x) = \int e^x \left\{ \frac{1}{\sqrt{x^2 + 1}} + \frac{-x}{(x^2 + 1)^{3/2}} \right\} dx$$

$$= \frac{e^x}{\sqrt{x^2 + 1}} + K$$

$$R(0) = 0 \Rightarrow K = -1$$

$$\therefore R(x) = \frac{e^x}{\sqrt{x^2 + 1}} - 1$$

$$\therefore R(1) = \frac{e}{\sqrt{2}} - 1$$

89. Answer (1)

$$\therefore I = \int_0^1 e^{1+2+3+\dots+n} dx$$

$$= e^{\frac{n(n+1)}{2}}$$

90. Answer (1)

$$m^2 + n^2 = 1 + 2 \cos(\alpha - \beta)$$

$$\therefore \max(m^2 + n^2) = 3 \text{ and } \min(m^2 + n^2) = -1$$

$$\therefore \text{Difference} = 4$$

