TEST - 7 (Paper - I)

ANSWERS

CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(D)	24.	(B)	47.	(B)
2.	(A)	25.	(D)	48.	(C)
3.	(A)	26.	(C)	49.	(A)
4.	(D)	27.	(B)	50.	(C)
5.	(D)	28.	(B)	51.	(D)
6.	(B)	29.	(B)	52.	(A)
7.	(A)	30.	(B)	53.	(B)
8.	(A, B, C)	31.	(B, D)	54.	(A, B, D)
9.	(A, C)	32.	(A, D)	55.	(B, C, D)
10.	(A, B, C, D)	33.	(C, D)	56.	(A, B, C)
11.	(A, D)	34.	(A, B)	57.	(B, C)
12.	(A)	35.	(C)	58.	(D)
13.	(B)	36.	(C)	59.	(C)
14.	(B)	37.	(B)	60.	(B)
15.	(B)	38.	(D)	61.	(B)
16.	(C)	39.	(A)	62.	(A)
17.	(4)	40.	(3)	63.	(5)
18.	(3)	41.	(1)	64.	(3)
19.	(4)	42.	(8)	65.	(1)
20.	(1)	43.	(3)	66.	(4)
21.	(6)	44.	(5)	67.	(3)
22.	(2)	45.	(2)	68.	(2)
23.	(3)	46.	(5)	69.	(3)

ANSWERS & HINTS

PART - I (CHEMISTRY)

1. Answer (D)

   $KMnO_4$ is of purple voilet and I_2 is of voilet colour.

2. Answer (A)

   $N_2O_4 + H_2O \rightarrow HNO_2 + HNO_3$

3. Answer (A)

   $Cu^{+2} + H_2S \rightleftharpoons CuS + 2H^+$

4. Answer (D)

   Cyclic complexes (chelates) are more stable than open chain complexes. Higher the number of cyclic rings, higher is the stability.

5. Answer (D)

   Fact.

6. Answer (B)

   In $CuCl_5^{-3}$ sp^3d hybridisation is present.

7. Answer (A)

   Fact.

8. Answer (A, B, C)

   $2Co(SCN)_2 + Hg^{+2} \rightarrow \underset{\text{Blue coloured ppt.}}{Co[Hg(SCN)_4]} + Co^{+2}$

9. Answer (A, C)

   $HBrO_4$ is stronger oxidising agent than $HClO_4$ and HIO_4.

10. Answer (A, B, C, D)

    $PCl_5 + H_2O \rightarrow H_3PO_4$

    $PH_3 + O_2 \rightarrow H_3PO_4$

    $P_4O_{10} + H_2O \rightarrow H_3PO_4$

11. Answer (A, D)

    In solid PCl_5 exists as $[PCl_4^+]$ $[PCl_6]^-$.

12. Answer (A)

    AgCl insoluble in hot water.

13. Answer (B)

    Barium show green flame test.

14. Answer (B)

    Precipitate of Al^{+3} form white gelatinous precipitate with NaOH.

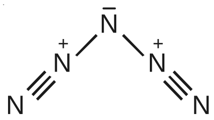
15. Answer (B)

    Cupellation method is used for purification of silver.

16. Answer (C)

    For van-Arkel method iodide should be volatile and thermally less stable and for Monds, carbonyl should be volatile and thermally less stable.

17. Answer (4)



18. Answer (3)

Solution O_2, SO_3, ClO_2 can convert PCl_3 to $POCl_3$.

19. Answer (4)

20. Answer (1)

Promethium (Pm) is radioactive

21. Answer (6)

Fact.

22. Answer (2)

It exists as $[ClO_2^+][ClO_4]^-$.

23. Answer (3)

6-metal-metal bonds are present.

PART - II (MATHEMATICS)

24. Answer (B)

$$I=\int\frac{\left(\sqrt{x}\right)^5}{\left(\sqrt{x}\right)^7+x^6}dx=\int\frac{dx}{\left(\sqrt{x}\right)^2+\left(\sqrt{x}\right)^7}=\int\frac{dx}{\left(\sqrt{x}\right)^7\left\{\frac{1}{\left(\sqrt{x}\right)^5}+1\right\}}$$

Put $\frac{1}{\left(\sqrt{x}\right)^5}+1=t \;\Rightarrow\; \frac{dx}{\left(\sqrt{x}\right)^7}=\frac{-2}{5}dt$

$$\therefore\quad I=\frac{-2}{5}\int\frac{dt}{t}=\frac{-2}{5}\log_e|t|+C$$

$$\therefore\quad I=\frac{-2}{5}\ln\left|\frac{1}{\left(\sqrt{x}\right)^5}+1\right|+C$$

$$\therefore\quad I=\frac{2}{5}\log_e\left\{\frac{x^{5/2}}{x^{5/2}+1}\right\}+C$$

$$\Rightarrow\quad \lambda=\frac{2}{5},\; a=\frac{5}{2}$$

$$\Rightarrow\quad a+\lambda=\frac{2}{5}+\frac{5}{2}=\frac{29}{10},\;\left\{\frac{29}{10}\right\}=\frac{9}{10}$$

25. Answer (D)

$\because \quad f'(x) = (4a-3)(1) + (a-7)\cos x$

$$= \left\{\frac{4a-3}{a-7} + \cos x\right\}\{a-7\}$$

$f'(x) \neq 0$ for non-existence of critical points

$\therefore \quad \frac{4a-3}{a-7} > 1$ or $\frac{4a-3}{a-7} < -1 \qquad \{\because \ -1 \le \cos x \le 1\}$

$\Rightarrow \quad \frac{3a+4}{a-7} > 0$ or $\frac{5a-10}{a-7} < 0$

$\therefore \quad a \in \left(-\infty, \frac{-4}{3}\right) \cup (7, \infty)$ or $a \in (2, 7)$

hence $a \in \left(-\infty, \frac{-4}{3}\right) \cup (2, \infty) \qquad \{\because$ at $a = 7$, $f'(x) \neq 0\}$

26. Answer (C)

$\because \quad [x^3]$ is not continuous and differentiable at integral points so $f(x)$ is continuous and differentiable in [4, 6] if

$$\left[\frac{(x-2)^3}{a}\right] = 0 \ \Rightarrow \ a \ge 64$$

27. Answer (B)

Divide Nr and Dr by $e^{1/x}$

$$\lim_{x\to 0^+} \frac{(1+a^3)e^{-1/x}+8}{e^{-1/x}+(1-b^3)} = 2 \ \Rightarrow \ \frac{0+8}{0+1-b^3} = 2 \ \Rightarrow \ b^3 = -3$$

$\Rightarrow \quad b = -3^{1/3}$

$$\lim_{x\to 0^-} \frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}}$$

$1 + a^3 = 2$

$a^3 = 1$

$a = 1$

28. Answer (B)

Put $x = t^6$

$\therefore \quad dx = 6t^5 dt$

$$I = \int \frac{6t^5}{t^3+t^4}dt = 6\int \frac{t^2}{t+1}dt$$

$$= 6\int\left\{t-1+\frac{1}{1+t}\right\}dt = 3t^2 - 6t + 6\ln|1+t| + C$$

$$I = 3(x)^{1/3} - 6(x)^{1/6} + 6\ln\left|1+\sqrt[6]{x}\right| + C$$

29. Answer (B)

Since given function is continuous but not-differentiable at $x = 0$ and $Lf'(0) = -3$, $Rf'(0) = 2$

$\Rightarrow \quad x = 0$ is a point of local minima and $f(0) = 1$

There is no other critical point.

Now, $f(-2) = 11$, $f(2) = 4 + \cos 2$

On comparing values of function at $x = -2, 0, 2$

global maximum occurs at $x = -2 \Rightarrow f(-2) = 11$

$\therefore$ option (B) correct

30. Answer (B)

Case-I

If $x > 0$, $|x| = x$

$$\therefore \int |x| \ln |x| dx$$

$$= \int x \ln x\, dx = \frac{x^2}{2} \ln x - \int \frac{1}{x} \frac{x^2}{2} dx$$

$$= \frac{x^2}{2} \ln |x| - \frac{x^2}{4} + C$$

Case-II

If $x < 0$, $|x| = -x$

$$\therefore \int |x| \ln |x| dx = -\int x \ln(-x) dx$$

$$= -\left\{ \ln(-x) \frac{x^2}{2} - \frac{x^2}{4} \right\} + C$$

$$= \frac{-x^2}{2} \ln |x| + \frac{x^2}{4} + C$$

Combining both option (B) correct

31. Answer (B, D)

Put $x^{3/2} = t \Rightarrow \frac{3}{2} x^{1/2} dx = dt$

$$\therefore \quad I = \frac{2}{3} \int \frac{dt}{\sqrt{1-t^2}} = \frac{2}{3} \sin^{-1} t + C = \frac{2}{3} \sin^{-1}\left(x^{3/2}\right) + C$$

32. Answer (A, D)

Slope of 1st curve = $2x + a$

at (1, 0) $\quad 2 + a = m_1$ (let)

Slope of 2nd curve = $c - 2x$

at (1, 0) $\quad c - 2 = m_1$ (let)

For touch $m_1 = m$

$\Rightarrow 2 + a = c - 2 \qquad \ldots$(i)

(1, 0) lies on both curves

$\Rightarrow 0 = 1 + a + b$ and $0 = c - 1 \qquad \ldots$(ii)

From (i) and (ii)

$\Rightarrow a = -3, b = 2, c = 1$

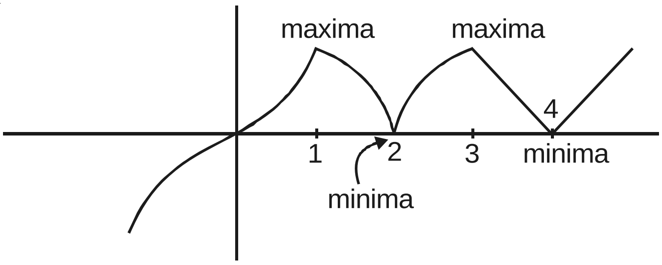
33. Answer (C, D)

$$\lim_{x\to 0}\left\{\frac{a^x+b^x+c^x}{3}\right\}^{\frac{\lambda}{x}}=e^{\lim_{x\to 0}\left\{\frac{a^x+b^x+c^x-3}{3}\right\}\frac{\lambda}{x}}$$

$$=e^{\frac{\lambda}{3}\lim_{x\to 0}\left\{\left(\frac{a^x-1}{x}\right)+\left(\frac{b^x-1}{x}\right)+\left(\frac{c^x-1}{x}\right)\right\}}$$

$$=e^{\frac{\lambda}{3}\ln(abc)}=(abc)^{\frac{\lambda}{3}}$$

34. Answer (A, B)



35. Answer (C)

Let $p = \sin^{m-1}x\cos^{n+1}x$

$$\therefore\ \frac{dp}{dx}=-(n+1)\sin^m x\cos^n x+(m-1)\sin^{m-2}x\cos^{n+2}x$$

On integrating both sides

$$I_{m,n}=-\left\{\frac{\sin^{m-1}x\cdot\cos^{n+1}x}{(n+1)}\right\}+\frac{(m-1)}{(n+1)}I_{m-2,n+2}$$

here $f(m,n)=\dfrac{m-1}{n+1}\ \Rightarrow\ f(2,3)=\dfrac{1}{4}$

36. Answer (C)

Let $P = x^{m+1}\{a+bx^n\}^p$

$$\therefore\ \frac{dP}{dx}=x^{m+1}\cdot p\{a+bx^n\}^{p-1}\{bnx^{n-1}\}+(a+bx^n)^p(m+1)x^m$$

$$= bpn\cdot x^{m+n}\{a+bx^n\}^{p-1}+(m+1)x^m\cdot(a+bx^n)^p$$

Integrating both sides,

$$P = bpn\ I_{m+n,\,p-1}+(m+1)I_{m,\,p}$$

or $$I_{m,p}=\frac{x^{m+1}\cdot(a+bx^n)^p}{m+1}-\frac{bnp}{m+1}I_{m+n,\,p-1}$$

$$\therefore\ f(m,n,b,p)=\frac{bnp}{m+1}\ \Rightarrow\ f(1,2,3,4)=12$$

37. Answer (B)

$$I_n = \int \sin^{n-1}x\left\{e^{ax}\sin x\right\}dx$$

$$= \sin^{n-1}x\left\{\frac{e^{ax}}{1+a^2}(a\sin x - \cos x)\right\} - \frac{1}{1+a^2}\int (n-1)\sin^{n-2}x\cos x\left\{e^{ax}(a\sin x - \cos x)\right\}dx$$

$$= \frac{e^{ax}\sin^{n-1}x\{a\sin x - \cos x\}}{1+a^2} - \frac{(n-1)}{(1+a^2)}\int\left\{a\sin^{n-1}x\cos x - \sin^{n-2}x\left(1-\sin^2 x\right)\right\}dx$$

$$= \frac{e^{ax}\sin^{n-1}x\{a\sin x - \cos x\}}{1+a^2} - \left\{\frac{n-1}{1+a^2}\right\}a\frac{\sin^n x}{n} + \left\{\frac{n-1}{1+a^2}\right\}I_{n-2} - \left\{\frac{n-1}{1+a^2}\right\}I_n$$

or, $I_n\left(n+a^2\right) = e^{ax}\sin^{n-1}x(a\sin x - \cos x) - (n-1)\dfrac{a\sin^n x}{n} + (n-1)I_{n-2}$

or, $I_n = \dfrac{e^{ax}\sin^{n-1}x\{a\sin x - \cos x\}}{(n+a^2)} - \dfrac{(n-1)a\sin^n x}{n(n+a^2)} + \left(\dfrac{n-1}{n+a^2}\right)I_{n-2}$

here $A = \dfrac{(n-1)}{n(n+a^2)}a,\ B = \left\{\dfrac{n-1}{n+a^2}\right\}$

$\therefore\quad A+B = \dfrac{a\left\{n^2-1\right\}}{n\left\{n+a^2\right\}}$

38. Answer (D)

39. Answer (A)

Solution of Q.No. 38 & 39

$\because\quad \lim\limits_{x\to 0^+} f(x) = \text{finite}$

$\Rightarrow\quad \lim\limits_{h\to 0} f(0+h) = \text{finite}$

$\Rightarrow\quad \lim\limits_{h\to 0}\dfrac{\sin h + ae^h + be^{-h} + c\ln(1+h)}{h^3} = \text{finite}$

at $h \to 0$, Nr $\to a + b$, Dr $\to 0$

$\Rightarrow\quad a + b = 0$ …(i)

Now, $\lim\limits_{h\to 0}\dfrac{\sin h + ae^h - ae^{-h} + c\ln(1+h)}{h^3} = \text{finite}$

$\Rightarrow\quad \lim\limits_{h\to 0}\dfrac{\dfrac{\sin h}{h} + (-a)\dfrac{(e^{-h}-1)}{h} + \dfrac{c\ln(1+h)}{h}}{h^2} = \text{finite}$

at $h \to 0$, Nr $\to 1 + a + a + c$, Dr $\to 0$

$\Rightarrow\quad 1 + 2a + c = 0$ …(ii)

$\therefore\quad \lim\limits_{h\to 0}\dfrac{\sin h + ae^h - ae^{-h} - (1+2a)\ln(1+h)}{h^3} = \text{finite}$

By L'hospital's rule

$$\lim_{h\to 0}\frac{\cos h + ae^{h} + ae^{-h} - \dfrac{(1+2a)}{(1+h)}}{3h^2};\left\{\frac{0}{0}\text{ forms}\right\}$$

$$\Rightarrow \lim_{h\to 0}\frac{-\sin h + ae^{h} - ae^{-h} + \dfrac{1+2a}{(1+h)^2}}{6h}$$

$$\text{Nr} \to 0 \quad \Rightarrow \quad 0 + a - a + \frac{1+2a}{1} = 0$$

$$\Rightarrow \boxed{a = -\frac{1}{2}} \Rightarrow \boxed{b = \frac{1}{2}} \Rightarrow \boxed{c = 0} \Rightarrow \boxed{a+b+c=0}$$

Now, $\displaystyle\lim_{x\to 0^+} f(x) = \lim_{h\to 0}\frac{\sin h - \frac{1}{2}e^{h} + \frac{1}{2}e^{-h}}{h^2}$

$$= \lim_{h\to 0}\frac{\cos h - \dfrac{e^h}{2} - \dfrac{e^{-h}}{2}}{3h^2} = \lim_{h\to 0} - \frac{\cos h - \dfrac{e^h}{2} - \dfrac{e^{-h}}{2}}{6}$$

$$= \frac{-1}{3}$$

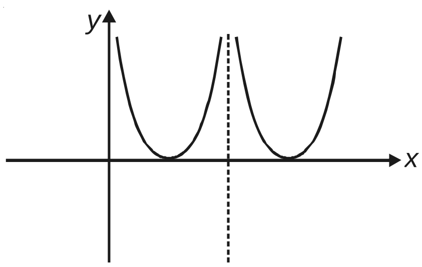
40. Answer (3)

$$K = \lim_{x\to\infty}\left[\int_0^x \frac{1}{\sqrt{1+t^2}}dt - \int_0^x \frac{1}{1+t}dt\right]$$

$$K = \lim_{x\to\infty}\left[\log\left\{x + \sqrt{1+x^2}\right\} - \log\{1+x\}\right]$$

$$\therefore \quad K = \lim_{x\to\infty}\left\{\log\left(\frac{x\left[1+\sqrt{\dfrac{1}{x^2}+1}\right]}{x\left\{1+\dfrac{1}{x}\right\}}\right)\right\} \Rightarrow K = \log_e 2 \Rightarrow e^K + 1 = 3$$

41. Answer (1)



Clearly f is not diff. at $x = \pi$

42. Answer (8)

$f'(x) = g(x)$

$f''(x) = g'(x) = -f(x)$

$\Rightarrow$ $f''(x) + f(x) = 0$

$\Rightarrow$ $f(x) = A\sin x$ or $A\cos x$

$f(2) = 4 = f'(1)$

$\Rightarrow$ f is constant

Now,

$$(f(2))^2 + (g(2))^2 = 4^2 + 4^2$$
$$= 32$$

$\therefore$ $\lambda = 8$

43. Answer (3)

$27^{\cos 2x} \cdot 81^{\sin 2x}$

$\Rightarrow$ $3^{\{3\cos 2x + 4\sin 2x\}}$

Now $-5 \le 3\cos 2x + 4\sin 2x \le 5$

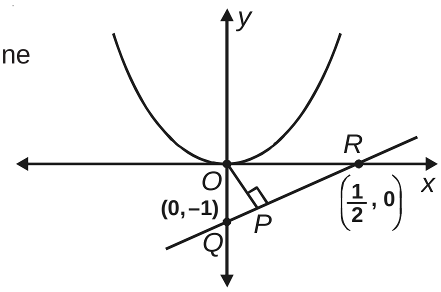
$\therefore$ Minimum value = 3^{-5}

$\Rightarrow$ $K = 3$

44. Answer (5)

Slope of curve, $\frac{dy}{dx} = 4x^3 + 6x + 2$

For nearest point on curve from line, tangent is parallel to line



$\therefore$ $4x^3 + 6x + 2 = 2$

$\therefore$ $x = 0$ or $x^2 = -\frac{3}{2}$ {not possible}

$\therefore$ $x = 0 \quad \Rightarrow \quad y = 0$

$\therefore$ Point nearest to line on curve (0, 0)

$\therefore$ Distance = $\left|\frac{-1}{\sqrt{1+(-2)^2}}\right| = \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{K}}$

$\Rightarrow$ $K = 5$

45. Answer (2)

$f(x) = e^{x^2/2}$

$$f(-x) = e^{(-x)^2/2} = e^{x^2/2}$$

and $\frac{f'(x)}{x} = \frac{e^{x^2/2} \cdot (2x/\lambda)}{x} = e^{x^2/2} \Rightarrow \frac{2}{\lambda} = 1 \Rightarrow \lambda = 2$

46. Answer (5)

Given $f(x) = ax^3 + bx^2 + 11x - 6$

from $f(1) = f(3)$

$a + b + 11 - 6 = 27a + 9b + 33 - 6$

$\Rightarrow$ $13a + 4b = -11$ …(i)

and $f'(x) = 3ax^2 + 2bx + 11$

$$2+\frac{1}{\sqrt{3}}=\frac{6+\sqrt{3}}{3}$$

Other root is $\frac{6-\sqrt{3}}{3}$

$$\frac{11}{3a}=\frac{36-3}{3}$$

$a = 1$

$b = -6$

$|a + b| = |1 - 6| = 5$

PART - III (PHYSICS)

47. Answer (B)

$$\tau=\vec{M}\times\vec{B}$$

$I\alpha = ia^2B$

$$\alpha=\frac{ia^2B}{I}$$

$$I=\frac{2M}{4}\left(\frac{a^2}{12}\right)+\frac{2M}{4}\left(\frac{a}{2}\right)^2$$

$$I=\frac{Ma^2}{6}$$

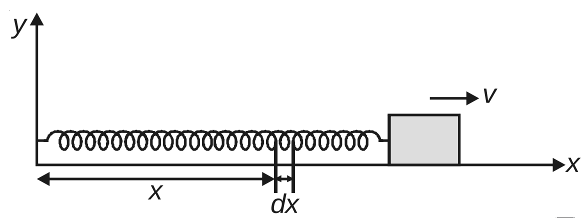
$$\therefore \quad \alpha=\frac{6ia^2B}{Ma^2}$$

$$=\frac{6iB}{M}$$

48. Answer (C)

Lets take an element dx of spring at distance x from origin

Velocity of this element is $v'=\frac{vx}{L}$



Magnetic force on this element is

$dF = dqv'B$

$$dF=\lambda_0 x^2\left(\frac{v}{L}x\right)Bdx$$

∴ Net force on the spring is

$$F = \frac{\lambda_0 vB}{L}\int_0^L x^3 dx$$

$$= \frac{\lambda_0 vBL^3}{4}$$

Force on the block = QvB

49. Answer (A)

$$z = \frac{\left(R - \frac{J}{\omega c}\right)(2\omega LJ)}{R + \left(2\omega L - \frac{1}{\omega c}\right)J}$$

$$z = \frac{\left(R - \frac{RJ}{2}\right)(RJ)}{R + \frac{R}{2}J}$$

$$z = \frac{\left(\frac{R^2}{2} + R^2 J\right)\left(R - \frac{R}{2}J\right)}{R^2 + \frac{R^2}{4}}$$

$$\Rightarrow \quad z = \frac{4R}{5} + \frac{3RJ}{5}$$

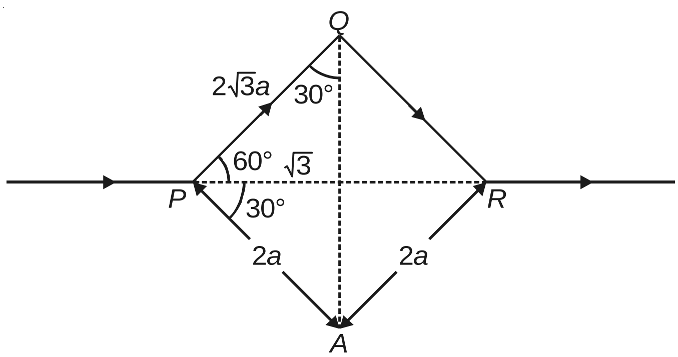
$|z| = R$

50. Answer (C)

$$B_1 = \frac{\mu_0 i}{4\pi a}(\sin 90° - \sin 60°)$$

$$B_2 = \frac{\mu_0 i}{4\pi(2a)}(\sin 60°)$$

$$B = (B_1 + B_2) \times 2 = \frac{\mu_0 i}{4\pi a}\left[2 - \frac{\sqrt{3}}{2}\right]$$



51. Answer (D)

Surface area of the body

$$S = 2\pi R^2 \left(\frac{R}{2R}\right)$$

$$= \pi R^2$$

Charge per unit area $\sigma = \frac{q}{\pi R^2}$ select a ring at an angle θ subtending an angle $d\theta$ at origin.

Radius of the ring = $R\cos\theta$

Thickness of the ring = $Rd\theta$

Area of the ring $dA = (2\pi R\cos\theta)Rd\theta$

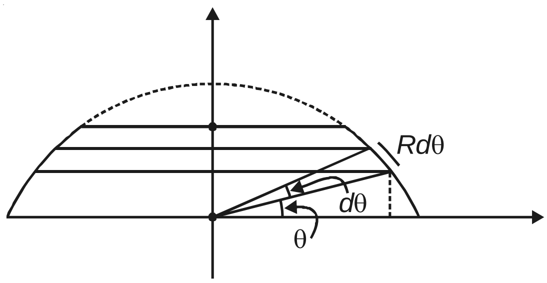
$\therefore$ Charge on the ring

$dq = \sigma dA$

$dq = 2\pi\sigma R^2\cos\theta d\theta$

$dq = \dfrac{2\pi q R^2\cos\theta d\theta}{\pi R^2}$

$= 2q\cos\theta d\theta$



Effective current di on the ring = $\dfrac{dq\omega}{2\pi}$

$$di = \frac{q\omega\cos\theta d\theta}{\pi}$$

Magnetic moment of the ring

$dM = \pi(R\cos\theta)^2 di$

$dM = R^2 q\omega\cos^3\theta d\theta$

$$M = R^2 q\omega\int_0^{\pi/6}\cos^3\theta d\theta$$

$$= \frac{R^2 q\omega}{4}\int_0^{\pi/6}(\cos 3\theta + 3\cos\theta)d\theta$$

$$M = \frac{11R^2 q\omega}{24}$$

52. Answer (A)

$$i = 3\sin\omega t + 4\sin\omega t\cos\frac{\pi}{3} + 4\cos\omega t\sin\frac{\pi}{3}$$

$$i = 5\sin\omega t + 2\sqrt{3}\cos\omega t$$

$$i_{max} = \sqrt{37}\text{ A}$$

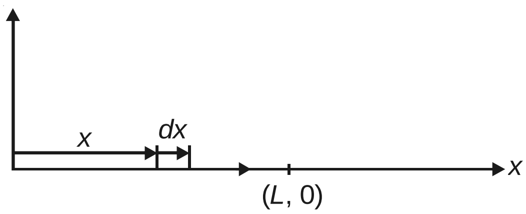
53. Answer (B)

Select a current element idx at a distance x from origin

Force on this element

$$dF = (idx)\hat{i}\times\left\{\frac{B_1 x}{L}\hat{j} + B_2\left(1+\frac{x^2}{L^2}\right)\hat{k}\right\}$$

$$dF = \frac{iB_1 x dx}{L}\hat{k} - iB_2\left(1+\frac{x^2}{L^2}\right)d\times\hat{j}$$

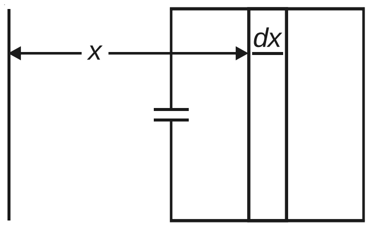


$$\therefore\quad F = \frac{iB_1L}{2}\hat{k} - \frac{4iB_2L}{3}\hat{j}$$

54. Answer (A, B, D)

Flux through the frame

$$\phi = \frac{\mu_0 i}{2\pi} 4\int_2^4 \frac{dx}{x}$$



$$\varepsilon = \left| -\frac{d\phi}{dt} \right| = \frac{2\mu_0 \ln 2\, i_0 \omega}{\pi} \cos\left(\omega t + \frac{\pi}{6}\right)$$

$$\varepsilon_{max} = \frac{2\mu_0 i_0 \omega \ln 2}{\pi}$$

$$Q_{max} = \frac{2\mu_0 i_0 \omega C \ln 2}{\pi}$$

at $t = 0$

$$\varepsilon = \frac{2\mu_0 \ln 2\, i_0 \omega}{\pi}\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore \quad Q = \frac{\sqrt{3}\mu_0 i_0 \omega C \ln 2}{\pi}$$

$$|i| = \left|\frac{dq}{dt}\right| = C\omega\varepsilon_0 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$\therefore \quad i_{max} = C\omega\varepsilon_0$$

$$= \frac{2C\omega^2 \mu_0 i_0 \ln 2}{\pi}$$

55. Answer (B, C, D)

Applying right hand thumb rule, it is clear that *A* is negatively charged while *B* is positively charged.

As $r = \frac{mv}{qB}$, so masses can be equal for different charges and vice-versa.

56. Answer (A, B, C)

Charge on the capacitor is given by

$$Q = Q_0 \cos\omega t \qquad \left(\omega = \frac{1}{\sqrt{LC}}\right)$$

$$\therefore \quad i = \frac{dQ}{dt} = -Q_0 \omega \sin\omega t$$

$$|i_{max}| = Q_0\omega \text{ at } t = \frac{\pi}{2\omega}$$

$$\therefore \quad t = \frac{\pi\sqrt{LC}}{2}$$

At $t = 0$

$i = -Q_0 \omega \sin 0°$

$i = 0$

when lower plate has charge Q_0, upper plate will have charge $-Q_0$

$\therefore \quad -Q_0 = Q_0\cos\omega t$

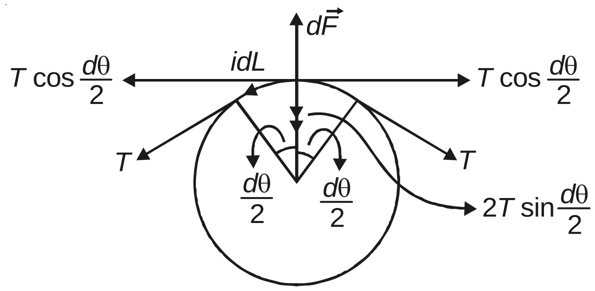
$\omega t = \pi$

$$t = \frac{\pi}{\omega}$$

$$t = \pi\sqrt{LC}$$

57. Answer (B, C)

The small segment subtending an angle $d\theta$ at centre is in equilibrium under the tension forces and Ampere's force $d\vec{F}$



$$\therefore \quad dF = 2T\sin\frac{d\theta}{2}$$

$$idLB = 2T\sin\frac{d\theta}{2} \approx Td\theta$$

$$\therefore \quad T = iB\frac{dL}{d\theta}$$

$$= iB\frac{Rd\theta}{d\theta}$$

$= iBR$

58. Answer (D)

$$z = \frac{\left(R_1 + \omega Lj - \frac{1}{\omega c}j\right)R_2}{(R_1+R_2)+\omega Lj - \frac{1}{\omega c}j}$$

$$\therefore \quad z = \frac{12-6\hat{j}}{5} \Rightarrow |z| = \frac{6\sqrt{5}}{5}\ \Omega$$

59. Answer (C)

$$(i_2)_{\max} = \frac{V_0}{R_2} = 1\text{ A}$$

60. Answer (B)

$$i = \frac{V_0}{|z|} = \sqrt{5}\text{ A}$$

61. Answer (B)

Magnetic moment of the wire

(1) Due to portion *AFED*

$$= (L)\left(\frac{3L}{2}\right)i\hat{k}$$

(2) Due to portion *DCBA*

$$\left(\pi\frac{L^2}{4} + \frac{L^2}{2}\right)i\hat{j}$$

$$\therefore \quad \vec{M} = \frac{(\pi+2)L^2}{4}i\hat{j} + \frac{3L^2}{2}i\hat{k}$$

62. Answer (A)

$$\vec{\tau} = \vec{M} \times \vec{B}$$

$$= \left(\frac{(\pi+2)}{4} L^2 i \,\hat{j} + \frac{3L^2 i\,\hat{k}}{2} \right) \times B_0 \hat{i}$$

$$= \frac{3L^2 i B_0 \hat{j}}{2} - \frac{(\pi+2)L^2 i B_0 \hat{k}}{4}$$

63. Answer (5)

$$U_{max} = \frac{1}{2} L (i_{max})^2$$

$$= \frac{1}{2} \times 10^{-4} \left(\frac{10}{250} \right)^2$$

$$= \frac{1}{2} \times 10^{-4} \left(\frac{10}{250} \right)^2$$

= 5 J

64. Answer (3)

$$\phi = \frac{\mu_0 i}{2\pi r} L^2$$

$$|E| = \left| -\frac{d\phi}{dt} \right| = \frac{\mu_0 L^2}{2\pi r} (2at)$$

$$i = \frac{\mu_0 L^2 at}{\pi r R}$$

65. Answer (1)

$$V_A - V_C = 0.5 \times 0.2 + 4 \times \frac{1}{2}$$

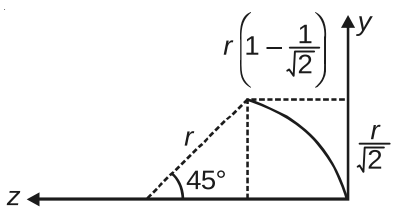
= 3 V

$\therefore \quad V_C$ = 1 V

66. Answer (4)

Radius of circular motion of the particle, $r = \dfrac{mV}{qB}$

Time-period of particle, $T = \dfrac{2\pi m}{qB_0}$



$$\therefore \quad t = \frac{33}{4} \frac{\pi m}{qB_0} = 4T + \frac{T}{8}$$

$$\therefore \quad z = -r \left(\frac{\sqrt{2} - 1}{\sqrt{2}} \right)$$

$$y = \frac{r}{\sqrt{2}}$$

distance from x-axis

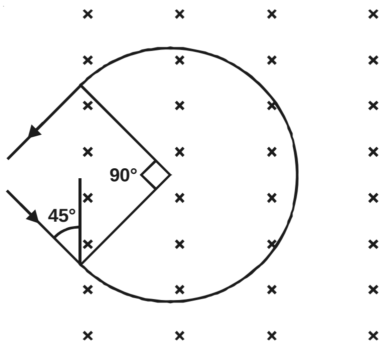
$$d = \sqrt{z^2 + y^2}$$

$$= \sqrt{\frac{2+1-2\sqrt{2}+1}{2}}r$$

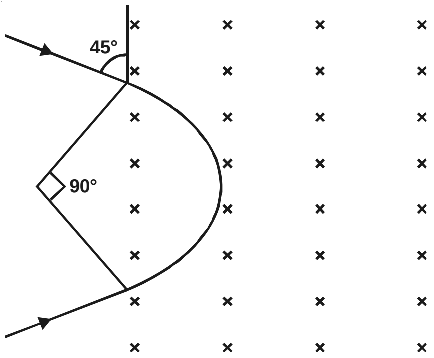
$$d = \sqrt{2-\sqrt{2}}\,r$$

67. Answer (3)

If the particle is positively charged



If the particle is negatively charged



$$\therefore \quad T_1 = \frac{2\pi r}{V}\left(\frac{270°}{360°}\right)$$

$$T_2 = \frac{2\pi r}{V}\left(\frac{90°}{360°}\right)$$

68. Answer (2)

$$\frac{L_1}{L_2} = \frac{2L+M}{2L-M} = 3$$

$$\Rightarrow \quad M = \frac{L}{2}$$

$\therefore \quad n = 2$

69. Answer (3)

At $t = 0$, $X_L = \infty$, $X_C = 0$

$$\therefore \quad i = \frac{9}{3} = 3 \text{ A}$$

❑ ❑ ❑

TEST - 7 (Paper - II)

ANSWERS

CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(C)	21.	(A)	41.	(C)
2.	(B)	22.	(A)	42.	(C)
3.	(D)	23.	(C)	43.	(A)
4.	(B)	24.	(A)	44.	(A)
5.	(B)	25.	(A)	45.	(A)
6.	(C)	26.	(A)	46.	(B)
7.	(D)	27.	(C)	47.	(A)
8.	(A)	28.	(A)	48.	(C)
9.	(A, B, D)	29.	(A, C)	49.	(A, B, C)
10.	(A, B, C, D)	30.	(A, B, D)	50.	(A, C, D)
11.	(B, D)	31.	(B, D)	51.	(A, B, C)
12.	(A, B, D)	32.	(A, D)	52.	(A, C)
13.	(5)	33.	(1)	53.	(5)
14.	(9)	34.	(3)	54.	(1)
15.	(0)	35.	(3)	55.	(3)
16.	(4)	36.	(1)	56.	(7)
17.	(4)	37.	(1)	57.	(1)
18.	(4)	38.	(5)	58.	(5)
19.	A → (p, s)	39.	A → (p, r)	59.	A → (q, t)
	B → (p, q, t)		B → (q)		B → (p)
	C → (q, r, t)		C → (t)		C → (p, r)
	D → (p)		D → (s)		D → (p, s)
20.	A → (p, r, s)	40.	A → (r)	60.	A → (p, r)
	B → (q, s)		B → (p)		B → (q, s)
	C → (r, s, t)		C → (p)		C → (q, r, t)
	D → (r, s, t)		D → (t)		D → (q, r, t)

ANSWERS & HINTS

PART - I (CHEMISTRY)

1. Answer (C)

   $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

   $HNO_2 \rightarrow HNO_3 + NO$

2. Answer (B)

   Fact.

3. Answer (D)

   Due to less back bonding

4. Answer (B)

   There will be geometrical isomer.

5. Answer (B)

   Due to symmetry.

6. Answer (C)

7. Answer (D)

   Due to high polarisation.

8. Answer (A)

   CuI is brown coloured due to charge transfer bond.

9. Answer (A, B, D)

10. Answer (A, B, C, D)

    KI, SF_4, Pt and H_2 can convert XeF_4 to Xe.

11. Answer (B, D)

12. Answer (A, B, D)

13. Answer (5)

    $\underset{(3)}{[Co(en)_2NO_2Br]^+}$ $\underset{(3)}{[Co(en)_2ONO\,Br]^+}$ $\underset{(3)}{[Co(en)_2BrCl]^+}$ $\underset{(3)}{[Co(en)_2NO_2Cl]^+}$ $\underset{(3)}{[Co(en)_2ONO\,Cl]^+}$

14. Answer (9)

    Fact.

15. Answer (0)

    $HgCl_2 + 2KI \rightarrow 2KCl + \underset{(Red)}{HgI_2}$

    $HgI_2 + 2KI \rightarrow K_2[HgI_4]$

    $K_2[HgI_4]$ is diamagnetic. Therefore, it does not have any unpaired electron.

16. Answer (4)

$SbCl_3 + H_2O \rightarrow SbOCl$

$BiCl_3 + H_2O \rightarrow BiOCl$

$CrF_6 \rightarrow CrO_2F_2 \rightarrow CrO_3$

$VCl_4 + H_2O \rightarrow VOCl_2$

17. Answer (4)

$[Fe(H_2O)_4U_2]\ ClH_2O$

18. Answer (4)

V_2O_5, SO_2, NO_2 and N_2O_3 are acidic.

19. Answer $\rightarrow$ A(p, s), B(p, q, t), C(q, r, t), D(p)

20. Answer $\rightarrow$ A(p, r, s), B(q, s), C(r, s, t), D(r, s, t)

PART - II (MATHEMATICS)

21. Answer (A)

$$I = \int \frac{x^2 - 1}{(x^2+1)\sqrt{x^4+1}}dx = \int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)\sqrt{x^2 + \frac{1}{x^2}}}dx$$

put $x + \frac{1}{x} = t \Rightarrow \left\{1 - \frac{1}{x^2}\right\}dx = dt$

$$\therefore \quad I = \int \frac{dt}{t\sqrt{t^2-2}} = \int \frac{t\,dt}{t^2\sqrt{t^2-2}}$$

put $t^2 - 2 = z^2 \Rightarrow 2t\,dt = 2z\,dz$

$$\therefore \quad I = \int \frac{z\,dz}{(z^2+2)z} \Rightarrow I = \frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{z}{\sqrt{2}}\right\} + C$$

$$\therefore \quad I = \frac{1}{\sqrt{2}}\tan^{-1}\left\{\frac{1}{\sqrt{2}}\sqrt{x^2 + \frac{1}{x^2}}\right\} + C$$

On comparison, $A = 4$

22. Answer (A)

Let $P(2t^2, 4t)$ be any point on parabola. Centre of circle is $O(0, -6)$ and radius is 1.

$\therefore \quad OP^2 = 4t^4 + (4t + 6)^2 = 4\{t^4 + 4t^2 + 9 + 12t\}$

$= 4x$, {Let $x = t^4 + 4t^2 + 12t + 9$}

$$\therefore \quad \frac{dx}{dt} = 4(t+1)(t^2 - t + 3) = 0$$

$\Rightarrow \quad t = -1$ {other roots are imaginary} and $\dfrac{d^2x}{dt^2} = 4\{3t^2 + 2\}$

$\Rightarrow \left.\dfrac{d^2x}{dt^2}\right|_{t=-1} > 0$

$\Rightarrow$ OP^2 is minimum at t = –1. But if A is any point on circle and on OP(min), then AP will be minimum, when OP is minimum as $AP = OP$ – {radius of circle}.

Thus required point is $P(2(-1)^2, 4(-1)) = (2, -4)$

23. Answer (C)

As $(1-x^4)^{\frac{7}{2}} = x^7\left\{\dfrac{1}{x^2} - x^2\right\}^{\frac{7}{2}}$

$\Rightarrow I = \displaystyle\int \frac{x^{-3}+x}{(x^{-2}-x^2)^{7/2}}dx$, put $x^{-2} - x^2 = t^2$

$\Rightarrow -2\{x^{-3} + x\}dx = 2t\,dt$

$\therefore I = -\displaystyle\int \frac{dt}{t^6} = \frac{1}{5t^5} + C$

$\therefore I = \dfrac{x^5}{5(1-x^4)^{5/2}} + C \Rightarrow K + \lambda = 5 + 5 = 10$

24. Answer (A)

$y = \dfrac{1}{2}\left[\dfrac{1}{x+1} + \dfrac{1}{x-1}\right]$

$\Rightarrow y_5(x) = (-1)^5\dfrac{5!}{2}\left\{\dfrac{1}{(x+1)^6} + \dfrac{1}{(x-1)^6}\right\}$

$\therefore -y_5(0) = (-1)^6\dfrac{5!}{2}\left\{\dfrac{1}{1} + \dfrac{1}{1}\right\} = 5!$

25. Answer (A)

$f(x) = \lim\limits_{n\to\infty}\dfrac{x^{1/n}-1}{\frac{1}{n}} = \lim\limits_{t\to 0}\dfrac{x^t-1}{t} = \log x \quad \left\{\text{put } \dfrac{1}{n} = t\right\}$

$\therefore f\left(\dfrac{1}{x}\right) = \log\left(\dfrac{1}{x}\right) = -\log x = -f(x)$

putting in given equation we get $K = 97$

26. Answer (A)

Let $f(x) = x^{\frac{1}{x}}$

$f(x) = e^{\frac{1}{x}\log x}$

$f'(x) = e^{\frac{1}{x}\log x}\left(\dfrac{1-\log x}{x^2}\right)$

Clearly $f'(x)$ is decreasing function for $x > C$

$\therefore$ $f(99) > f(100)$

$$99^{\frac{1}{99}} > 100^{\frac{1}{100}}$$

$\Rightarrow$ $99^{100} > 100^{99}$

Similarly

$200^{201} > 201^{200}$

$1000^{2000} > 2000^{1000}$

$500^{5000} > 5000^{500}$

27. Answer (C)

Put $x = \cos\theta$

$$\Rightarrow \quad u = \sec^{-1}\left\{\frac{1}{2\cos^2\theta - 1}\right\} = \sec^{-1}(\sec 2\theta) = 2\theta$$

and $y = \sqrt{1-x^2} = \sin\theta$

$$\therefore \quad u = 2\sin^{-1} y \Rightarrow \frac{du}{dy} = \frac{2}{\sqrt{1-y^2}} = \frac{2}{\sqrt{x^2}}$$

$$\Rightarrow \quad \left.\frac{du}{dy}\right|_{x=\frac{1}{2}} = 4$$

28. Answer (A)

$$I = \int \frac{dx}{x^{1/3}\{1+x^{1/6}\}}$$

put, $x^{1/6} = t$

$\Rightarrow$ $x = t^6$

$\Rightarrow$ $dx = 6t^5 dt$

$$\therefore \quad I = 6\int \frac{t^5 dt}{t^2(1+t)} = 6\int \frac{t^3}{t+1} dt$$

$$\Rightarrow \quad I = 6\int \frac{t^3+1-1}{t+1} dt$$

$$\therefore \quad I = 6\int \left\{(t^2 - t + 1) - \frac{1}{t+1}\right\} dt$$

$$= 6\left\{\frac{t^3}{3} - \frac{t^2}{2} + t - \log(1+t)\right\} + C$$

$$\therefore \quad I = 2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(1+x^{1/6}) + C$$

29. Answer (A, C)

$18y\frac{dy}{dx}=3x^2$

∴ normal makes equal intercepts on axes

∴ $\frac{-1}{dy/dx}=\pm 1$ or $\frac{dy}{dx}=\pm 1$

∴ $6y(\pm 1) = x^2$ or $36y^2 = x^4$

or $4x^3 - x^4 = 0$

∵ $9y^2 = x^3$

∴ $x = 4,\ y=\pm\frac{8}{3}$ {$x = 0$ is rejected}

30. Answer (A, B, D)

Because of $|y|$, we redefine as

$y > 0 \Rightarrow y = \frac{1}{2}x \quad \therefore\ x \geq 0$

$y < 0 \Rightarrow y = \frac{1}{10}x \quad \therefore\ x < 0$

hence, $f(x)=y=\begin{cases}\frac{x}{10} & ;\ x<0\\ \frac{x}{2} & ;\ x\geq 0\end{cases}$

Clearly (A), (D) holds true

Also, $f(0 + h) = f(0 - h) = f(0) = 0$

∴ Continuous. It is not differentiable as $Rf'(0) = \frac{1}{2}$ and $Lf'(0) = \frac{1}{10}$

31. Answer (B, D)

$\frac{dy}{dx}=(x^2-1)^n\left[(x^2-1)(2x+1)+2x(n+1)(x^2+x+1)\right]$

∵ $f(x)$ has local extremum $\frac{dy}{dx}$ must change sign

i.e. $f(1^+)$ and $f(1^-)$ should be of opposite sign.

Now, we know that $(x^2 + x + 1)$ is always positive $(b^2 - 4ac) = D$ = negative and first term tends to zero as $x \Rightarrow 1$. Hence sign of $\frac{dy}{dx}$ will depend upon $(x^2 - 1)^n$.

If n is even, then $f(1^+)$ and $f(1^-)$ will have same sign but they must be of opposite sign for existence of extremum. Hence n must be odd.

32. Answer (A, D)

$$Y-y=\left\{\frac{-1}{dy/dx}\right\}(X-x)$$. It meets x-axis, $y = 0$ at G.

$\therefore$ $G\left(y\frac{dy}{dx}+x, 0\right)$. Given $OG = \pm 2x$

$\therefore$ $y\frac{dy}{dx}+x=\pm 2x$

or $y\,dy = x\,dx$ or $y\,dy = -3x\,dx$

Integrating ;

$$\frac{y^2}{2}=\frac{x^2}{2}-\frac{a^2}{2} \quad \text{or} \quad \frac{y^2}{2}=\frac{-3x^2}{2}+\frac{K^2}{2}$$

$$\underbrace{x^2-y^2=a^2}_{\text{Hyperbola}} \quad \text{or} \quad \underbrace{3x^2+y^2=K^2}_{\text{Ellipse}}$$

33. Answer (1)

$$\lim_{x\to 0^-}(1+[x])^{\frac{-1}{x+[x]}}=\lim_{h\to 0}(1+[0-h])^{\frac{-1}{0-h+[0-h]}}$$

$$=\lim_{h\to 0}(1-1)^{\frac{-1}{-h-1}}=\lim_{h\to 0}(0)^{\frac{1}{h+1}}=0=a$$

$\Rightarrow$ $4a + 1 = 1$

34. Answer (3)

$$\lim_{x\to\infty}\left(1+\frac{2a}{x^2}+\frac{4}{x^4}\right)^{x^2+x+1}=e^6$$

$$\therefore \quad e^{\lim\limits_{x\to\infty}(x^2+x+1)\left\{1+\frac{2a}{x^2}+\frac{4}{x^4}-1\right\}}=e^6$$

$$\therefore \quad e^{\lim\limits_{x\to\infty}(x^2+x+1)\left\{\frac{2a}{x^2}+\frac{4}{x^4}\right\}}=e^6$$

$$\therefore \quad e^{\lim\limits_{x\to\infty}\left\{2a+\frac{2a}{x}+\frac{2a}{x^2}+\frac{4}{x^2}+\frac{4}{x^3}+\frac{4}{x^4}\right\}}=e^6$$

$\therefore$ $e^{2a} = e^6$

$\Rightarrow$ $a = 3$

35. Answer (3)

$$\lim_{x\to 0^-}\left\{a+\frac{x^3}{3^x}\right\}=a+0\,;\,f(0)=1$$

$$\lim_{x\to 0^+}\left\{(1+\sin x)^{\frac{1}{\sin x\cos x}}-\sqrt{e^b}+1\right\}=e^{\lim\limits_{x\to 0^+}\left(\frac{\sin x}{\sin x\cos x}\right)}-\sqrt{e^b}+1$$

$= e^1 - e^{b/2} + 1 = 1$; $\{\because\ a = 1\}$

$\therefore$ $b = 2$

$\Rightarrow$ $a + b = 3$

36. Answer (1)

Let $1 - x = t^2$ $\quad \therefore \quad I = -2\int (1-t^2)^2 dt$

$\therefore \quad dx = -2t\,dt$ $\quad = -2\int (1+t^4-2t^2)dt$

$$= -2\left[t + \frac{t^5}{5} - \frac{2t^3}{3}\right]$$

$$= -2t\left[\frac{15+3t^4-10t^2}{15}\right]$$

$$= -\frac{2}{15}\sqrt{1-x}\left\{3x^2+4x+8\right\}$$

$\Rightarrow \quad P = \frac{-2}{15} \Rightarrow 15P+3=1$

37. Answer (1)

$$f(x) = \int_{\pi/4}^{x} 4(\sin t - 3\cos t)dt$$

$$f'(x) = 4\sin x - 3\cos x > 0 \;\forall\; x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$$

$\therefore$ at $n = \frac{\pi}{2}$, $f(x)$ has maxima

$$f(\pi/2) = \int_{\pi/4}^{\pi/2} (4\sin t - 3\cos t)dt$$

$$= \left[-4\cos t - 3\sin t\right]_{\pi/4}^{\pi/2}$$

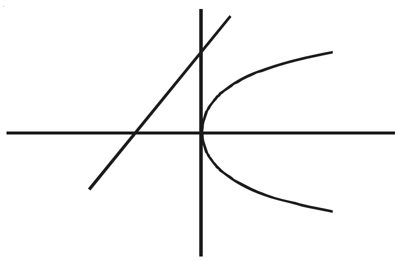
$$= \left(\frac{7}{\sqrt{2}} - 3\right)$$

38. Answer (5)

Minimum distance will be along common normal slope of normal

to line and parabola is = $\frac{-1}{2}$

Point of parabola is



$$\left(4\cdot\frac{1}{2^2}, -2\cdot 4\cdot\frac{-1}{2}\right)$$

(1, 4)

$\therefore \quad \alpha + \beta = 5$

39. Answer $\rightarrow$ A(p, r), B(q), C(t), D(s)

(A) $I=\int \frac{dx}{x^n(1+x^n)^{1/n}}=\int \frac{dx}{x^{n+1}\left\{\frac{1}{x^n}+1\right\}^{1/n}}$

put $\frac{1}{x^n}+1=t$

$\Rightarrow \quad \frac{-n}{x^{n+1}}dx=dt$

$\therefore \quad I=\frac{-1}{n}\int \frac{dt}{t^{1/n}}=\frac{1}{(1-n)}(t)^{1-\frac{1}{n}}+C$

$\therefore \quad I=\frac{1}{1-n}\{x^{-n}+1\}^{1-\frac{1}{n}}+C=\frac{x^{(1-n)}}{(1-n)}\{1+x^n\}^{1-\frac{1}{n}}+C$

(B) $I=\int \frac{dx}{x^2\cdot x^{n-1}\left\{1+\frac{1}{x^n}\right\}^{\left(\frac{n-1}{n}\right)}}=\int \frac{dx}{x^{n+1}\left\{1+\frac{1}{x^n}\right\}^{\left(\frac{n-1}{n}\right)}}$

put $\frac{1}{x^n}+1=t$

$\frac{-n}{x^{n+1}}dx=dt$

$\therefore \quad I=\frac{-1}{n}\int \frac{dt}{t^{\left(\frac{n-1}{n}\right)}}$

$\therefore \quad I=-\left\{1+\frac{1}{x^n}\right\}^{1/n}+C=-(1+x^{-n})^{1/n}+C$

(C) $I=\int \frac{e^x dx}{\sqrt{e^{2x}+1}}$

put $e^x = t$

$e^x dx = dt$

$\therefore \quad I=\int \frac{dt}{\sqrt{t^2+1}}=\ln\left\{t+\sqrt{t^2+1}\right\}+C$

$\therefore \quad I=\ln\left\{e^x+\sqrt{e^{2x}+1}\right\}+C$

(D) $I=\int \frac{dx}{(e^{2x}+1)^2}=\int \frac{e^x dx}{e^x(e^{2x}+1)^2}$

put $e^x = \tan\theta$

$e^x dx = \sec^2\theta\, d\theta$

$\therefore \quad I=\int \frac{\sec^2\theta\, d\theta}{\tan\theta \sec^4\theta}=\int \frac{d\theta}{\tan\theta \sec^2\theta}$

$I=\int \frac{(1-\sin^2\theta)\cos\theta}{\sin\theta} d\theta$

put $\sin\theta = t$

$\cos\theta\, d\theta = dt$

$\therefore \quad I=\int \frac{(1-t^2)}{t} d\theta = \ln(\sin\theta)-\frac{\sin^2\theta}{2}+C$

$\therefore \quad I = x-\frac{1}{2}\ln\{1+e^{2x}\}+\frac{1}{2}\left\{\frac{1}{e^{2x}+1}\right\}+C$

40. Answer $\rightarrow$ A(r), B(p), C(p), D(t)

(A) $\lim_{x\to 0}\frac{\sin(\ln(1+x))}{\ln(1+\sin x)}=\lim_{x\to 0}\frac{\sin(\ln(1+x))}{\ln(1+x)}\cdot\frac{\sin x}{\ln(1+\sin x)}\cdot\frac{\ln(1+x)}{x}\cdot\frac{x}{\sin x}$

$= 1\cdot1\cdot1\cdot1 = 1$

(B) $\lim_{x\to 0}\frac{x\left\{\frac{2\tan x}{1-\tan^2 x}\right\}-2x\tan x}{(2\sin^2 x)^2}=\lim_{x\to 0}\frac{2x\tan x(\tan^2 x)}{(1-\tan^2 x)4\sin^4 x}$

$=\lim_{x\to 0}\frac{\frac{2x}{x}\left\{\frac{\tan x}{x}\right\}^3}{(1-\tan^2 x)4\left(\frac{\sin x}{x}\right)^4}=\frac{2\cdot1\cdot(1)^3}{(1-0)\cdot4\cdot(1)^4}=\frac{1}{2}$

(C) $\lim_{x\to 2}\left\{\left[\frac{x(x+2)(x-2)}{(x-2)(x^2+2x+4)}\right]^{-1}-\left[\frac{\sqrt{x}\left(\sqrt{x}+\sqrt{2}\right)}{x-2}-\frac{\sqrt{2}\left(\sqrt{x}+\sqrt{2}\right)}{x-2}\right]^{-1}\right\}$

$=\lim_{x\to 2}\left\{\frac{x^2+2x+4}{x(x+2)}-1\right\}=\lim_{x\to 2}\frac{4}{x(x+2)}=\frac{4}{2\cdot4}=\frac{1}{2}$

(D) $\lim_{x\to\infty}\frac{(10)^n\left\{\left(\frac{1}{10}\right)^n-1\right\}}{(10)^{n+1}\left\{\left(\frac{1}{10}\right)^{n+1}+1\right\}}=\frac{(0-1)}{10(0+1)}=\frac{-1}{10}=\frac{-2}{20}$

$\Rightarrow \quad \alpha = 2$

PART - III (PHYSICS)

41. Answer (C)

$$\int_0^{VA} dV = \int_0^{R/4} Bw.xdx$$

$$V_A - V_0 = \frac{BwR^2}{32}$$

$$\int_{V_0}^{V_B} dV = \int_0^{3R/4} Bwxdx \Rightarrow \frac{9}{32}BwR^2 = V_B - V_0$$

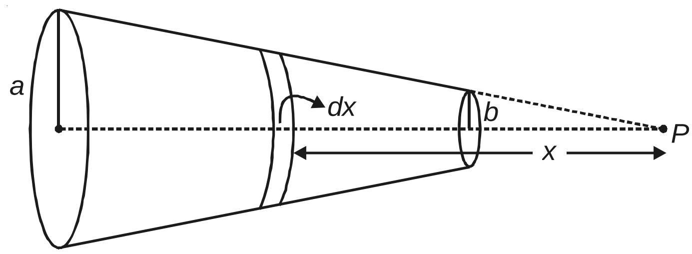
$$V_B - V_0 = \frac{9}{32}BwR^2$$

$$V_A - V_0 = \frac{BwR^2}{32}$$

$$(V_B - V_0) - (V_A - V_0) = \left(\frac{9}{32} - \frac{1}{32}\right)BwR^2 = \frac{1}{4}BwR^2$$

42. Answer (C)

Consider a ring of thickness *dx* at distance of *x* from point *P*.



Radius of the ring

$$y = \frac{a}{L}x$$

Magnetic field due to this ring at point *P* is

$$dB = \frac{\mu_0 i n dx y^2}{2(y^2 + x^2)^{3/2}}$$

$$dB = \frac{\mu_0 i n a^2 x^2 dx}{2L^2\left(\frac{a^2}{L^2} + 1\right)^{3/2} x^3}$$

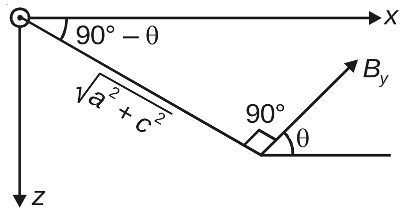
∴ Field due to whole coil is equal to

$$B = \frac{\mu_0 i n a^2 L}{2(a^2 + L^2)^{3/2}} \int_{\frac{bL}{a}}^{L} \frac{dx}{x}$$

$$= \frac{\mu_0 i n a^2 L}{2(a^2 + L^2)^{3/2}} \ln\frac{a}{b}$$

43. Answer (A)

Direction of magnetic field due to wire kept a long *y*-axis is as shown in figure.



$$\vec{B}_y = \frac{\mu_0 i_2}{2\pi\sqrt{a^2+c^2}}\cos\theta\hat{i} - \frac{\mu_0 i_2 \sin\theta\hat{k}}{2\pi\sqrt{a^2+c^2}}$$

$$\therefore \quad \vec{B}_y = \frac{\mu_0 i_2 c}{2\pi(a^2+c^2)}\hat{i} - \frac{\mu_0 i_2 a}{2\pi(a^2+c^2)}\hat{k}$$

Similarly, magnetic field due to wire kept along x-axis.

$$\vec{B}_x = -\frac{\mu_0 i_1 c}{2\pi(b^2+c^2)}\hat{j} + \frac{\mu_0 i_1 b}{2\pi(b^2+c^2)}\hat{k}$$

44. Answer (A)

From 0 – 3 s

$$i = \left|\frac{d\theta}{Rdt}\right| = \frac{5}{4} = 1.25 \text{ A}$$

For 3 s to 4 s

$$i = \left|\frac{d\theta}{Rdt}\right| = 0$$

Similarly for 4 to 8 s

i = 0.625 A

and for 8 to 10 s

i = 0

45. Answer (A)

Lets take small element dx on the ring

Charge on this element

$$dQ = \frac{Q}{2\pi R}dx$$

Force on this element

$dF = dQ \times E_{in}$

$\therefore$ torque of this field

$d\tau = RdF$

$= RdQE_{in}$

Total torque

$$\tau = \frac{R\theta}{2\pi R}\int (E_{in})dx$$

$$= \frac{RQ}{2\pi R} \times \left(-\pi R^2 \frac{dB}{dt}\right)$$

$$\tau = \frac{Q}{2\pi} \times \pi R^2 \times B_0 \times 2t$$

$$= Q \cdot B_0 R^2 t = mR^2\alpha$$

$$\alpha = \frac{QB_0 t}{m} = \frac{d\omega}{dt}$$

$$\omega = \frac{QB_0 t^2}{2m}$$

46. Answer (B)

Mass of frame of wire

$$m = \left(3L + \frac{2L}{\pi}\right)\mu$$

$$I = \frac{[3\pi + 2]\mu g}{2B}$$

$$\therefore \quad Bi\frac{2L}{\pi} = \mu\left(3L + \frac{2L}{\pi}\right)g$$

47. Answer (A)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

at ω = 1.2 ω_0

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \sqrt{R^2 + \left(\frac{6\omega_0 L}{5} - \frac{5}{6\omega_0 C}\right)^2}$$

$$= \sqrt{R^2 - \frac{L}{C}\left(\frac{121}{900}\right)}$$

48. Answer (C)

$$i_{average} = \frac{\int i\,dt}{\int dt}$$

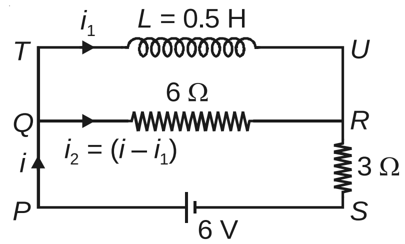
$$= \frac{\pi(0.5)^2}{4} + \frac{(0.5)(0.5)}{2}$$

$$= \left(\frac{\pi + 2}{16}\right) \text{A}$$

49. Answer (A, B, C)

Applying *KVL* in loop *PQRS*

$6 - 6(i - i_1) - 3i = 0$ …(i)



and in loop *PTQS*

$$6 - L\frac{di_1}{dt} - 3i = 0 \quad \text{…(ii)}$$

From (i) and (ii)

$$6 - \frac{1}{2}\left(\frac{di_1}{dt}\right) - 3\left(\frac{6+6I_1}{9}\right) = 0$$

$$\Rightarrow \quad -\frac{dI_1}{I_1 - 2} = 4dt + C$$

$t = 0$, $I_1 = 0$

$$\therefore \quad I_{1(t)} = 2\left[1 - e^{-4t}\right]$$

$$I_{(t)} = 2 - \frac{4}{3}e^{-4t}$$

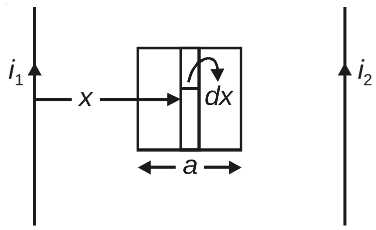
$$I_{1(t)} = 2\left[1 - e^{-4t}\right]$$

50. Answer (A, C, D)

Magnetic flux through the square loop due to i_1

$$\phi_1 = \frac{\mu_0 i_1 a}{2\pi}\int_a^{2a}\frac{dx}{x}$$

$$= \frac{\mu_0 i_1 a \ln 2}{2\pi}$$



Similarly

$$\phi_2 = \frac{\mu_0 i_2 a}{2\pi}\ln 2$$

$$|\phi_{\text{Nett}}| = \frac{\mu_0 a \ln 2}{2\pi}(i_2 - i_1) = \frac{\mu_0 a \log_e 2}{2\pi} \times (2t^2)$$

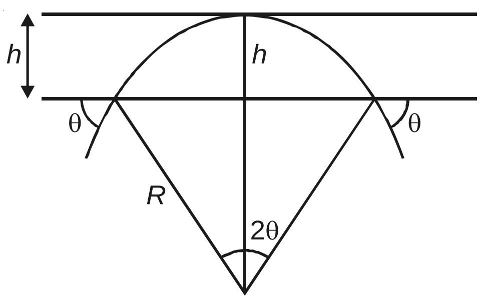
$$|e| = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0 a \ln 2}{2\pi}(4t)$$

$$\therefore \quad e = \frac{\mu_0 a \ln 2t}{\pi}$$

51. Answer (A, B, C)

$h = R(1 - \cos\theta)$

$$R = \frac{h}{1 - \cos\theta}$$



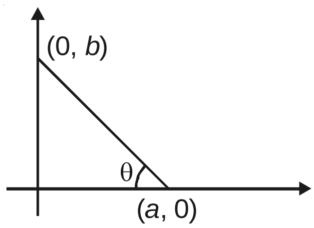
$$\Rightarrow \quad \frac{mv}{QB} = \frac{h}{1 - \cos\theta}$$

$$v = \frac{\theta h B}{m(1 - \cos\theta)}$$

Angle of deviation = 2θ

52. Answer (A, C)

Magnitude of Ampere's force on *AB*



$$|F_m| = iB\sqrt{a^2+b^2}\sin\theta$$

$$|F_m| = iBb$$

Similarly, magnitude of Ampere's force on *BC*

$= 2iBb$

Magnetic moment of the loop = i × (Area of the loop)

$$= i\pi\left(\frac{ab}{4}+\frac{ab}{8}\right)$$

$$= \frac{3i\pi ab}{8}\hat{k}$$

53. Answer (5)

$$P = \frac{E_0 I_0 R}{2\left[(X_L - X_C)^2 + R^2\right]^{1/2}}$$

$$= \frac{4E_0 I_0}{2\left[(7-4)^2 + 4^2\right]^{1/2}}$$

$$= \frac{2E_0 I_0}{5}$$

54. Answer (1)

$$Z = \frac{V_0}{I_0} = \sqrt{2}$$

$$\cos\frac{\pi}{4} = \frac{R}{Z} = \frac{1}{\sqrt{2}}$$

∴ $R = 1\ \Omega$

55. Answer (3)

$\phi_2 = Mi_1$

$$\Rightarrow \quad M = \frac{\phi}{i}$$

$$= \frac{450}{150} = 3\text{ H} = 3\text{ ohm}\times\text{s}$$

56. Answer (7)

$$B_1 = \frac{2\mu_0 i}{2R} = \frac{\mu_0 i}{R}$$

$$B_2 = \frac{3\mu_0 i}{4R}$$

$$\therefore \quad \frac{B_1}{B_2} = \frac{4}{3}$$

57. Answer (1)

$$\frac{E_1}{E_2}=\frac{\frac{BLV}{2}}{BLV\sin 30^\circ}=1$$

58. Answer (5)

For the portion of wire in *xy* plane

$$\overrightarrow{M_1}=i\left(\frac{3}{5}\times\frac{20}{3}\times\frac{1}{2}\right)\hat{k}$$

$$=4\hat{k}$$

For the portion of wire in *xz* plane

$$\overrightarrow{M}_2=i\left(\frac{1}{2}\times 5\times\frac{3}{5}\right)\hat{j}$$

$$=3\hat{j}$$

$$\left|\overrightarrow{M}_1+\overrightarrow{M}_2\right|=\sqrt{4^2+3^2}$$

= 5 A-m^2

59. Answer → A(q, t), B(p), C(p, r), D(p, s)

$Z_A = X_C$

$$Z_B=\sqrt{R^2+X_C^2}$$

$$Z_C=\sqrt{R^2+(X_L-X_C)^2}$$

$$Z_D=\frac{1}{\sqrt{\frac{1}{R^2}+\left(\frac{1}{X_L}-\frac{1}{X_C}\right)^2}}$$

∴ On decreasing R, Z_B, Z_L and Z_D will decrease.

Average power is zero in pure inductive circuit.

On increasing frequency, phase angle will increase in (B), (C) and (D).

60. Answer → A(p, r), B(q, s), C(q, r, t), D(q, r, t)

Direction of magnetic field can be found right hand thumb rule.

Magnetic field in the proximity of infinitely long current carrying conductor tends to infinity and therefore discontinuous.

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